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# Stratified Density Gravity: Gravitation Without Fundamental Time

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## Abstract

We present Stratified Density Gravity (SDG), a reformulation of gravitation in which the fundamental manifold is described by (x,y,z,ρ) rather than (t,x,y,z), with ρ representing local gravitational depth / energy density. In this framework time is not a fundamental coordinate; instead, it emerges as a macroscopic ordering parameter associated with irreversible relaxation of density stratification (entropy production). We show that curvature is sourced by spatial stratification of ρ and we obtain a field equation in which local curvature depends on inhomogeneities in ρ, while large-scale curvature depends on the homogeneous background component of ρ. The local sector reproduces Newtonian gravity in the weak-field limit, which fixes the relevant coupling. The global sector yields a dynamical large-scale curvature term that provides an alternative to the cosmological constant Λ and explains cosmic acceleration as a background-density phase. Because curvature depends on second spatial derivatives of ρ, singularities do not form in collapsed regions: high density cores remain finite. The framework therefore preserves the known weak-field and observational successes of General Relativity (GR) while addressing the cosmological constant problem and the classical singularity problem.

**Keyword:** Modified gravity, General relativity, Cosmology, Cosmic acceleration, Black holes, Singularity resolution, Horizonless compact objects, Scale-dependent growth, Bianchi identity, Stratified density field

## Introduction

General Relativity (GR) models' gravity as the curvature of a four-dimensional space-time manifold with coordinates (t, x, y, z) [1,2]. GR has been confirmed in a broad range of regimes, including perihelion precession, gravitational redshift, light bending and lensing and the existence of gravitational waves [3]. These classical and modern tests of General Relativity are comprehensively reviewed in [4].

However, several open issues remain:

- **Cosmological constant problem.** The observed accelerated expansion of the universe is typically modeled in GR by introducing a cosmological constant Λ, interpreted as a vacuum stress-energy with effective equation-of-state parameter $w \approx -1$ [5-7,8]. The magnitude of Λ is not predicted by GR and naive estimates of the vacuum energy overshoot the observed effective value by many orders of magnitude. This historical development and its modern interpretation as dark energy are reviewed in [8].
- **Spacetime singularities.** Classical GR predicts curvature singularities in black-hole interiors and at the Big Bang. At these points the manifold description formally breaks down.
- **Time and irreversibility**. In GR, time t is placed on geometric equal footing with space. But physically, time exhibits an arrow: macroscopic processes proceed irreversibly, entropy increases and causal ordering is asymmetric. GR by itself does not explain the origin of this arrow.

In this work we develop a framework in which density, not time, plays the role of the "fourth" coordinate of the gravitational manifold. We call this Stratified Density Gravity (SDG). The basic principles are:

- The physical manifold is (x, y, z, ρ), where ρ is a scalar coordinate that encodes local gravitational depth / energy density.
- Time is not fundamental; it is interpreted as an emergent macroscopic parameter that orders the irreversible relaxation of ρ-stratification and the associated increase in entropy.
- Curvature is sourced directly by spatial stratification of ρ. The field equations separate naturally into a local inhomogeneity term and a homogeneous background term.
- The local coupling is fixed by requiring that the weak-field limit reproduces Newtonian gravity. The global coupling is fixed by requiring consistency with homogeneous cosmology; this replaces the role of Λ with a quantity determined observationally, rather than inserted by hand.
- Because curvature is controlled by second derivatives of ρ, not by a divergent stress- energy, extremely dense regions become finite and smooth at their core. The r→ 0 singularities of GR do not occur.

We will show that this construction:

- Reproduces Newtonian gravity and the standard weak-field phenomenology of GR (e.g. gravitational redshift, lensing).

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Innovative Journal of Applied Science

- Produces late-time cosmic acceleration through a dynamical background-density coupling rather than a fundamental cosmological constant.
- Eliminates curvature singularities at r = 0 in static, spherically symmetric configurations.
- And provides a geometric–thermodynamic origin for macroscopic time's arrow.

We argue that SDG preserves the empirical successes of GR while addressing three core conceptual gaps: the cosmological constant problem, singularity formation and the absence of an intrinsic arrow of time [5-7].

## Foundational Assumptions

### Spatial extension is three-dimensional

Physical extension is described by spatial coordinates (x, y, z).

### Density is the fourth coordinate

We extend the manifold not by t but by a scalar ρ,

$(x^i)$ = (x, y, z, ρ), i = 1, 2, 3, 4,

where ρ is interpreted physically as gravitational depth / energy density. The manifold is therefore four-dimensional, but its fourth coordinate is density, not time.

In the weak-field regime we will identify

$$\rho \approx \frac{\emptyset}{c^2}, \qquad (1)$$

where Φ is the Newtonian gravitational potential and c is the speed of light. This normalization makes ρ dimensionless to leading order and ties it directly to an experimentally accessible potential.

### Curvature is generated by stratification of ρ

Gravity is described as curvature induced by spatial gradients and second derivatives of ρ. Mass-energy does not act through an externally specified stress-energy tensor $T_{\mu\nu}$; instead, it appears in the theory through the spatial structure of the scalar density coordinate ρ.

### Time is emergent from ρ-relaxation

Macroscopic "time flow" and its arrow are associated with the irreversible smoothing of initially steep ρ-gradients. Entropy production corresponds to the redistribution and relaxation of stratified density. Observers parameterize this monotonic relaxation using a scalar parameter, which they call *t*. In this sense, *t* is emergent and thermodynamic, not fundamental and geometric.

### No singularities

Because we will formulate curvature in terms of second derivatives of ρ, rather than unbounded stress-energy sources, collapsed objects approach a high but finite value $\rho_c$ with $\Delta\rho \to 0$ and $\Delta_i\Delta_j\rho \to$ finite at r = 0. This prevents curvature blow-up. The same mechanism eliminates a formal "Big Bang" singularity: the early universe can begin at a high but finite ρ, without infinite curvature.

## Geometric Structure of the $(x, y, z, \rho)$ Manifold

### Metric ansatz

We define the line element on the manifold M with coordinates $(x^i)$ = (x, y, z, ρ) as

$$ds^2 = g_{\alpha\beta}(x, y, z, \rho)dx^\alpha dx^\beta + f(\rho)d\rho^2, \qquad \alpha, \beta \in \{x, y, z\}, \qquad (2)$$

where $g_{\alpha\beta}$ is the spatial three-metric on constant-ρ slices and $f(\rho)\, d\rho^2$ measures the "geometric separation" between density layers.

High-field meaning and range of ρ. In weak fields ρ = $\Phi/c^2$ is dimensionless and small. In strong fields, ρ is a geometric depth coordinate: constant-ρ slices foliate the manifold; increasing ρ labels deeper layers. The physically relevant range is set by the solution of (18) given matter sources and boundary data. In compact objects, ρ saturates at a finite central value $\rho_c$ with $\nabla\rho \to 0$, while in cosmology $\rho_{bg}$ is the homogeneous component that controls the sign of $\beta\rho_{bg}$ (Sec. 6–7). No singular behaviour is required or allowed: ρ and its second derivatives remain finite in all regular solutions.

We define the Christoffel symbols $\Gamma_{ij}^k$ in the usual way and compute the Riemann tensor $R_{ij\ell}^k$, Ricci tensor $R_{ij}$, Ricci scalar R and the Einstein tensor

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}, \qquad i, j \in \{x, y, z, \rho\}. \qquad (3)$$

Physically, $f(\rho)d\rho^2$ plays the role that the gravitational potential depth plays in GR's $g_{tt}$ component: deeper density layers correspond to slower local processes and stronger curvature. In standard GR this behaviour is often described in terms of "time dilation" due to $g_{tt}$; here it is recast geometrically along the ρ direction itself.

## Field Equation

We postulate the field equation of SDG in its general form:

$$G_{ij} = \alpha\nabla_i\nabla_{j\rho} + \beta\rho g_{ij,} \qquad i, j \in \{x, y, z, \rho\}. \qquad (4)$$

Here $\nabla_i$ is the covariant derivative compatible with $g_{ij}$. The two couplings are:

- **α:** controls local curvature sourced by inhomogeneities in ρ. This term must reduce to Newtonian gravity in the weak-field limit and reproduce standard GR tests [2,3].
- **β:** controls homogeneous background curvature sourced by the spatially uniform part of ρ. This term will reproduce the role that Λ plays in GR cosmology but in SDG we will determine β from cosmological observables rather than insert it as a constant [5,6].

In what follows, α is determined by the Newtonian limit and β is determined by homogeneous cosmology. Throughout, we write $\rho_{bg}$ for the large-scale homogeneous back- ground of $\rho$ (the cosmological mean).

## Local Limit and the Determination of α

To recover Newtonian gravity, consider the weak-field, slow-motion limit around nonrelativistic matter. Let Φ be the Newtonian gravitational potential, which satisfies

$$\nabla^2\Phi = 4\pi G\rho_m, \quad (5)$$

with $\rho_m$ the usual mass density. In this regime we identify

$$\rho = \frac{\Phi}{c^2}\ , \quad (6)$$

which makes ρ dimensionless to leading order. Taking the Laplacian of (6) gives

$$\nabla^2\rho = \frac{4\pi G}{c^2}\rho_m. \quad (7)$$

We now evaluate (4) in this weak-field regime. On Solar System scales the homogeneous background term $\beta\rho g_{ij}$ is negligible compared to local inhomogeneities, so we drop it. We also assume the spatial metric is nearly Euclidean so covariant derivatives reduce to ordinary derivatives.

Under these assumptions, take the spatial trace of (4). Let $\alpha, \beta \in \{x, y, z\}$ denote spatial indices. Then

$$G^{\alpha}_{\alpha} \simeq \alpha\nabla_{\alpha}\nabla^{\alpha}\rho = \alpha\nabla^2\rho. \quad (8)$$

In standard GR, the weak-field 00-component of Einstein's equations effectively repro- duces (5) and gives

$$G^{\alpha}_{\alpha} \simeq \frac{8\pi G}{c^2}\rho_m, \quad (9)$$

see, e.g., [2]. Using (7) in (8) yields

$$\alpha\left(\frac{4\pi G}{c^2}\rho_m\right) = \frac{8\pi G}{c^2}\rho_m,$$

so that

$$\alpha = 2. \quad (10)$$

Thus the coefficient multiplying $\nabla_i\nabla_j\rho$ is fixed by the Newtonian limit and the weak field phenomenology of GR [3]. It is not a tunable parameter.

With this, the SDG field equation becomes

$$G_{ij} = 2\nabla_i\nabla_j\rho + \beta\ \rho\ g_{ij}. \quad (11)$$

## Bianchi identity and the differential constraint for $\rho$

The covariant Bianchi identity $\nabla^i G_{ij} = 0$ applied to the SDG field equation

$$G_{ij} = 2\ \nabla_i\nabla_j\rho + \beta\rho g_{ij} \quad (12)$$

implies a consistency condition that constrains permissible ρ-configurations. Taking $\nabla^i$ of (12) and using $\nabla^i g^{ij} = 0$ gives

$$0 = 2\nabla^i\nabla_i\nabla_j\rho + \nabla_j(\beta\rho). \quad (13)$$

Commuting covariant derivatives on a scalar gradient yield (for any scalar $\varphi$) $\nabla^i\nabla_i\nabla_j\varphi = \nabla_j(\square\varphi) + R^k_j\nabla_k\varphi$, where $\square \equiv \nabla^i\ \nabla_i$. Thus (13) becomes the vector identity

$$\nabla_j(2\ \square\rho + \beta\rho) = -2R^k_j\nabla_k\rho\ . \quad (14)$$

Equation (14) is the differential constraint on $\rho$ implied by the Bianchi identity. Two important limits follow immediately.

### Homogeneous (FRW) background

On Hubble scales we assume $\rho\ =\ \rho_{bg}$ is spatially homogeneous, so $\nabla_k\rho_{bg} = 0$. Then the right-hand side of (14) vanishes and we obtain

$$\nabla_j(2\square\rho_{bg} + \beta\rho_{bg}) = 0 \Rightarrow 2\square\rho_{bg} + \beta\rho_{bg} = C(\lambda)\ , \quad (15)$$

with C(λ) a (spatially constant) integration function along the monotone evolution parameter λ that orders the macroscopic relaxation (cf. Sec. 8). In an exactly stationary background one may set C = const.; in practice C(λ) encodes slow secular drift of the background density sector.

### Weakly curved, static configurations

For the static, spherically symmetric case of Sec. 9 with ρ = ρ(r) and small curvature near the center, the term $R^k_j\nabla_{k\rho}$ is subleading. Thus (29) reduces to

$$\nabla_j(2\square\rho + \beta\rho) \approx 0 \qquad \Rightarrow 2\ \square\rho + \beta\rho \approx const. \quad (16)$$

which reproduces the radial structure equation used in Sec. 9 and yields the regular near-core expansion

$$\rho(r) \simeq \rho_c\left[1 - \frac{\beta}{12}\ r^2 + O(r^4)\right]. \quad (17)$$

This demonstrates explicitly that the SDG source $2\nabla_i\nabla_j\rho + \beta\rho g_{ij}$ is compatible with $G_{ij} = 0$, ensuring consistency of the field equations and forming the basis for the background evolution and perturbation dynamics developed later. Equation (14) is the precise statement that the SDG source $2\nabla_i\nabla_j\rho + \beta\rho g_{ij}$ is compatible with $\nabla_i G_{ij} = 0$. It will be used below to produce background evolution equations and the linear perturbation dynamics.

## Matter coupling and covariance of the $\rho$ equation

In the Newtonian/weak-field limit (Sec. 5) we identified $\rho = \Phi/c^2$ and recovered $\nabla^2\rho = \left(\frac{4\pi G}{c^2}\right)\rho_m$. The covariant generalization consistent with this limit and with (14) is to close the system by a single scalar equation

$$\rho = \left(\frac{4\pi G}{c^2}\right)S - \left(\frac{\beta}{2}\right)\rho + U'(\rho). \quad (18)$$

where $S$ is a covariant scalar that reduces to $\rho_m$ for nonrelativistic matter (pressure $p \ll \rho_m c^2$) and U(ρ) is an optional self-interaction potential encoding high-density microphysics. A minimal, conservative choice is

$$S = \rho_m - \frac{3p}{c^2}, \quad (19)$$

i.e. the usual relativistic trace combination that reduces to $\rho_m$ in the weak-field regime.

Consistency with the Bianchi constraint. Taking $\nabla_j$ of (18) and substituting into (14) gives

$\nabla_j(2\,\Box\rho+\beta\rho)=\frac{8\pi G}{c^2}\nabla_j S+2\nabla_j U'(\rho)=-2R_j^k\nabla_k\rho,$

which is automatically satisfied in homogeneous FRW (where $\nabla_j = 0 = \nabla_j\rho$) and reduces, in static weak fields, to the Poisson form used in Sec. 5.

**Interpretation**

Equations (12) and (18) together define SDG as a closed system: curvature is determined by spatial stratification of $\rho$, while $\rho$ is determined by matter (through $S$), the background coupling β (Sec. 6) and possible self-interaction $U$. Setting $U \equiv 0$ is sufficient for all results shown in this paper; nonzero $U$ can encode finite-density microphysics without altering the large-scale conclusions.

## Consistency with Local and Weak-Field Tests

A complete gravitational framework must reproduce all weak-field phenomena verified by general relativity. Here we summarize the consistency of Stratified Density Gravity (SDG) with the three canonical tests: Gravitational redshift, light deflection and gravitational- wave propagation.

### Gravitational redshift

In SDG, redshift arises from gradients in the density potential ρ. For a static weak field, the temporal component of the metric satisfies $g_{00} \simeq 1+2\Phi\rho/c^2$, where $\Phi_\rho = (\alpha G(\rho-\rho_0)/r^2dr)$ dr plays the role of the Newtonian potential. The fractional frequency shift between two density layers is therefore

$$\frac{\Delta\nu}{\nu}\approx-\frac{\Delta\Phi_\rho}{c^2}$$

which coincides with the general relativistic prediction to first order when α = 2 as derived in Section 5. This ensures that all laboratory and solar-gravitational redshift tests (e.g. Pound–Rebka, Hafele–Keating, GPS corrections) are automatically satisfied.

**Redshift as a constraint on the SDG clock factor**

In SDG the physical proper time along a worldline is related to the global evolution parameter λ through

$$d\tau \;=\; F(\rho)\,d\lambda, \tag{20}$$

where $F(\rho)$ is a priori an arbitrary positive function reflecting that time is not a fundamental coordinate but an emergent rate associated with stratification. Gravitational redshift measurements, however, fix the functional dependence of $F(\rho)$.

Consider two static observers located at radii $r_1$ and $r_2$ in a weak gravitational field. Both follow integral curves of the density foliation, so their rate of proper time with respect to λ is given by $F(\rho_1)$ and $F(\rho_2)$ respectively. If n(λ) counts wave crests of a photon emitted at $r_1$ and received at $r_2$, then

$$\nu_{emit}=\frac{dn}{d\tau_1}=\frac{1}{F(\rho_1)}\frac{dn}{d\lambda},\;\nu_{rec}=\frac{dn}{d\tau_2}=\frac{1}{F(\rho_2)}\frac{dn}{d\lambda}, \tag{21}$$

so that the observable frequency ratio satisfies

$$\frac{\nu_{rec}}{\nu_{emit}}=\frac{F(\rho_1)}{F(\rho_2)}. \tag{22}$$

In general relativity the redshift between two static observers in a static metric

$$ds^2=g_{tt}(r)c^2dt^2+g_{rr}dr^2+r^2d\Omega^2 \tag{23}$$

is given exactly by

$$\frac{\nu_{rec}}{\nu_{emit}}=\sqrt{\frac{g_{tt}(r^2)}{g_{tt}(r^1)}}. \tag{24}$$

Because laboratory, satellite and solar-system redshift measurements agree with (24) to parts in $10^6$– $10^7$, SDG must satisfy

$$\frac{F(\rho^1)}{F(\rho^2)}=\sqrt{\frac{g_{tt}(r^2)}{g_{tt}(r^1)}}\text{ for all weak-field configurations.} \tag{25}$$

The only solution of this functional equation is

$$F(\rho)=\frac{C}{\sqrt{g_{tt}^{eff}(\rho)}}, \tag{26}$$

where C is a constant absorbed by rescaling λ. Choosing units such that C = 1 yields the SDG clock law

$$d\tau=\frac{d\lambda}{\sqrt{g_{tt}^{eff}(\rho)}}, \tag{27}$$

identical in form to the GR relation $d\tau=\sqrt{g_{tt}}dt$ but with the role of coordinate time t played by the emergent parameter λ. Expanding (27) for a weak gravitational potential where $g_{tt}=1+2\Phi/c^2+O(\Phi^2)$ and using $\rho\;=\;\Phi/c^2$ (Sec. 5) gives

$$F(\rho)=1-\rho+O(\rho^2), \tag{28}$$

which reproduces the observed gravitational time dilation to leading and next-leading post-Newtonian orders.

Thus, gravitational redshift does not merely constrain SDG—it uniquely fixes the functional dependence of $F(\rho)$ and ties the rate of emergent time directly to the effective lapse function of the metric. There is no remaining freedom in the definition of $d\tau$ consistent with experiment.

### Light deflection and time delay

In SDG, photons follow null geodesics of the same curved spatial geometry determined by ρ(x). The metric near a mass distribution can be written in isotropic coordinates as

$$ds^2\simeq\left(1+\frac{2\Phi_\rho}{c^2}\right)c^2dt^2-\left(1-\frac{2\Phi_\rho}{c^2}\right)(dx^2+dy^2+dz^2)$$

which leads to a light-bending angle

$$\Delta\theta=\frac{4GM}{c^2b},$$

identical to that predicted by GR when α = 2. This equality extends to the Shapiro time delay in radar ranging experiments, confirming that SDG reproduces post-Newtonian optics with the same first-order parameter γ = 1.

**Parametrized post-Newtonian parameters**

The Parametrized Post-Newtonian (PPN) framework expands the metric around Minkowski space in powers of $U/c^2$, where U is the

Newtonian potential generated by a slowly moving source [9]. In isotropic coordinates the weak-field metric of any metric theory can be written as

$$g_{00} = 1 - \frac{2U}{c^2} + 2\beta_{PPN}\frac{U^2}{c^4} + O\left(\frac{U^3}{c^6}\right), \tag{29}$$

$$g_{0i} = O\left(\frac{v}{c^3}\right). \tag{30}$$

$$g_{ij} = -\left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij} + O\frac{U^2}{c^4}. \tag{31}$$

as in the standard PPN formalism [9].

The parameter γ measures the amount of spatial curvature generated per unit Newtonian potential, while $\beta_{PPN}$ quantifies nonlinear self-gravity.

In SDG the weak-field limit is governed by

$$G_{ij} = 2\nabla_i\nabla_j\rho, \tag{32}$$

where the Newtonian limit (Sec. 5) fixes $\rho = U/c^2$ and α = 2. Expanding $g_{ij}$ to first order and using the standard linearized expressions for the Einstein tensor in harmonic gauge [10],

$$G_{00}^{(2)} = 2\nabla^2 U/c^2, \tag{33}$$

$$G_{ij}^{(2)} = \frac{2}{c^2}\partial_i\partial_j U + \frac{2}{c^2}\delta_{ij}\nabla^2 U, \tag{34}$$

the SDG source term

$$2\nabla_i\nabla_j\rho = \frac{2}{c^2}\partial_i\partial_j U \tag{35}$$

implies that the spatial metric perturbation must satisfy

$$h_{ij}^{(2)} = 2\frac{U}{c^2}\delta_{ij}, \tag{36}$$

which corresponds to the PPN value

$$\gamma = 1. \tag{37}$$

This is consistent with the equality of the SDG and GR predictions for light deflection and the Shapiro time delay derived above.

To determine $\beta_{PPN}$, we extend $G_{00}$ to second order. In the PPN expansion one finds [9]

$$G_{00} = 2\frac{\nabla^2 U}{c^2} + 2(2\beta_{PPN} - 1)\frac{U\nabla^2 U}{c^4} + O\left(\frac{U^3}{c^6}\right). \tag{38}$$

In SDG the identification $\rho = U/c^2$ ensures that all nonlinear contributions to $G_{00}$ generated by $\nabla_i\nabla_j\rho$ appear in exactly the same combinations as in GR: There is no additional scalar degree of freedom and no modification of the second-order gravitational self-energy.

Thus, the coefficient of $U\nabla^2 U$ must match the GR value, yielding

$$\beta_{PPN} = 1. \tag{39}$$

Hence SDG reproduces the full suite of post-Newtonian constraints:

$$\gamma = 1, \beta_{PPN} = 1, \tag{40}$$

in agreement with all present Solar-System bounds. Together with the exact Newtonian limit (α = 2), gravitational redshift and light-deflection results above, this shows that SDG is locally indistinguishable from GR throughout all presently tested weak-field regimes.

## Gravitational-wave propagation

Linearizing the field equation $G_{ij} = \alpha\nabla_i\nabla_{j\rho} + \beta_{\rho g ij}$ about a homogeneous background $\rho = \rho_0 + \delta_\rho, g_{ij} = \eta_{ij} + h_{ij}$ and imposing the transverse–traceless gauge gives, to first order,

$$\Box h_{ij} = 0,$$

up to corrections of order $\nabla_i\nabla_j\,(\delta_\rho/\rho_0)$, which vanish in vacuum. Thus, in empty space where $\nabla_\rho \simeq 0$, SDG predicts the same two tensorial polarizations of gravitational waves traveling at c as GR. This satisfies all constraints from LIGO/Virgo timing and polarization measurements and confirms that no additional degrees of freedom propagate in the linear limit.

**Summary**

The recovery of the exact Newtonian potential (α = 2), correct light deflection, standard redshift formula and tensorial gravitational waves implies that SDG is indistinguishable from GR in all presently tested weak-field regimes. Deviations are therefore expected only in the strong-field and cosmological sectors, precisely where GR requires ad-hoc constants or encounters singularities.

## Cosmological Limit and the Determination of β

We now consider the opposite regime: homogeneous cosmology.

Assume that on Hubble scales ρ is spatially homogeneous and slowly varying:

$$\rho \approx \rho_{bg} = \text{constant on large scales}, \tag{41}$$

so that $\nabla_i\nabla_j\rho \approx 0$ on those scales. Here $\rho_{bg}$ is the smooth background density of the universe. In this limit, (11) reduces to

$$G_{ij} \approx \beta\rho_{bg}g_{ij}. \tag{42}$$

In homogeneous, isotropic cosmology using GR, the Friedmann equation for a spatially flat FRW universe is [2,4,5,8]

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{bg} + \frac{\Lambda}{3}, \tag{43}$$

where H is the Hubble parameter, a is the scale factor and Λ is the cosmological constant. In GR, Λ appears in Einstein's equations as a term proportional to $g_{\mu\nu}$.

Comparing (42) with the GR form $G_{\mu\nu} = -\Lambda g_{\mu\nu}$ in the vacuum-energy-dominated limit suggests the identification

$$\Lambda_{eff} = -\beta\rho_{bg}. \tag{44}$$

This identification parallels the role of Λ in standard relativistic cosmology as discussed comprehensively in [4,8]. Eliminating $\Lambda_{eff}$ using (43) gives

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Innovative Journal of Applied Science

$$\Lambda_{eff} = 3H^2 - 8\pi G\,\rho_{bg}.$$

Combining with (44) yields

$$3H^2 - 8\pi G\rho_{bg} = -\beta\rho_{bg}.$$

Solving for β: $\beta = 8\pi G - \frac{3H^2}{\rho_{bg}}$ (45)

Equation (45) is the large-scale analogue of (10). It shows:

- β is not arbitrary. It is determined by the cosmological expansion rate H and the homogeneous background density $\rho_{bg}$.
- β can evolve in cosmic time, because H and $\rho_{bg}$ evolve.
- The role played by Λ in GR is here played by a dynamical curvature coupling β that emerges from the same ρ field that drives local gravity [5-7].

## Emergence of an effective cosmological term

Equation (24) shows that in a homogeneous and isotropic background the SDG field equation

$$G_{\mu\nu} = 2\nabla_\mu\nabla_\nu\rho_{bg} + \beta\rho_{bg}g_{\mu\nu} \quad (46)$$

reduces to a modified Friedmann system in which the quantity $\beta\rho_{bg}$ multiplies the metric exactly as a cosmological constant would in general relativity. We now make this correspondence explicit.

For a spatially flat FRW metric,

$$ds^2 = -c^2d\lambda^2 + a^2(\lambda)d\vec{x}^2, \quad (47)$$

the Einstein tensor satisfies

$$G_{00} = 3H^2, G_{ij} = -(2\dot{H} + 3H^2)a^2\delta_{ij}. \quad (48)$$

Because $\rho_{bg}$ depends only on λ, the terms involving second covariant derivatives simplify:

$$\nabla_0\nabla_0\rho_{bg} = \ddot{\rho_{bg}}, \qquad \nabla_i\nabla_j\rho_{bg} = H\dot{\rho}_{bg}\,a^2\,\delta_{ij}. \quad (49)$$

Using these relations in (46), the 00-component becomes

$$3H^2 = 2\ddot{\rho_{bg}} + \beta\rho_{bg}c^2 + \frac{8\pi G}{c^2}\,\rho_m, \quad (50)$$

where the matter term arises from identifying the trace part of the Einstein tensor with the usual GR coupling (Sec. 5). In the quasi-static regime in which $\ddot{\rho}_{bg}$ is small compared to $H\dot{\rho}_{bg}$— a condition satisfied at late cosmological times and encoded in Eq. (24)—the derivative term may be moved to the right-hand side of the equation. One then obtains the effective Friedmann equation

$$3H^2 = \frac{8\pi G}{c^2}\rho_m - \beta\rho_{bg}c^2. \quad (51)$$

Comparing (51) with the standard GR form

$$3H^2 = \frac{8\pi G}{c^2}(\rho_m + \rho_\Lambda), \quad (52)$$

we identify the SDG-induced effective dark-energy density as

$$\rho_{\Lambda,eff} = -\frac{\beta c^2}{8\pi G}\rho_{bg}. \quad (53)$$

Equivalently, the geometric term multiplying the metric in (46) behaves exactly as a cosmological constant of magnitude

$$\Lambda_{eff} = -\beta\rho_{bg}. \quad (54)$$

**Interpretation**

Equation (54) shows that cosmic acceleration in SDG originates not from a fundamental vacuum constant but from the large-scale stratification of the density coordinate. As the universe expands and $\rho_{bg}$ evolves, the effective cosmological term evolves with it, providing a natural mechanism for late-time acceleration without introducing a separate dark-energy sector. This replaces the cosmological constant problem of GR with a dynamical, physically motivated quantity tied directly to the geometry of stratification.

## Relation to the deceleration parameter

It is useful to connect β to an observable cosmological quantity. The deceleration parameter q is defined by

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2}. \quad (55)$$

For an FRW universe containing a perfect fluid with energy density ϵ and pressure p, one may write

$$q = \frac{1}{2}\left(1 + 3\frac{p}{\epsilon}\right). \quad (56)$$

Combining the standard Friedmann acceleration equation with (43) gives

$$3H^2(1+q) = 8\pi G\rho_{bg}. \quad (57)$$

Insert (57) into (45) to express β in terms of q:

$$\beta = 8\pi G - \frac{3H^2}{\rho_{bg}} = 8\pi G - \frac{8\pi G\rho_{bg}}{\rho_{bg}(1+q)} = 8\pi G - \frac{8\pi G}{1+q}.$$

Thus

$$\beta = \frac{8\pi G}{1+q}q. \quad (58)$$

Therefore $\beta$ and q have the same sign:

$$\beta < 0 \Leftrightarrow q < 0 \qquad \text{(accelerating expansion)}, \quad (59)$$

$$\beta > 0 \Leftrightarrow q < 0 \qquad \text{(decelerating expansion)}. \quad (60)$$

The transition β = 0 corresponds to q = 0, which is the moment at which the universe switches from deceleration to acceleration. In GR+Λ, the onset of acceleration is attributed to “dark energy dominance.” In SDG, it is a geometric phase change in the density coupling β.

## Normalization and dimensional consistency of the stratification field

A central requirement for internal consistency is that all terms appearing in the field equations carry the correct physical dimensions. In Stratified Density Gravity, the fundamental scalar field ρ is defined through the weak-field limit as

$$\rho \simeq \frac{\Phi}{c^2}, \quad (61)$$

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where Φ is the Newtonian gravitational potential. Since Φ has units of velocity squared, ρ is dimensionless. This choice ensures that $\rho$ encodes the relative depth of spacetime stratification rather than an energy density.

The field equation

$$G_{ij} = 2\nabla_i\nabla_{j\rho} + \beta\rho g_{ij} \quad (62)$$

therefore, requires the coefficient β to carry units of inverse length squared,

$$[\beta] = L^{-2}, \quad (63)$$

so that both terms on the right-hand side have the same physical dimension as the Einstein tensor.

In a homogeneous cosmological background, $\rho = \rho_{bg}(t)$ and the stratified term vanishes at leading order, leaving

$$G_{\mu\nu} \simeq \beta\rho_{bg}\, g_{\mu\nu}. \quad (64)$$

This identifies the effective cosmological curvature as

$$\Lambda_{eff} \equiv -\beta\, \rho_{bg}, \quad (65)$$

which carries the correct dimensions of inverse length squared and plays the same geometric role as the cosmological constant in Einstein gravity.

Importantly, no additional energy density is introduced: The quantity $\rho$ is dimensionless and all dimensional information resides in the single curvature scale β. This guarantees internal dimensional consistency and ensures that the theory does not introduce hidden or redundant degrees of freedom.

## Background closure from the Bianchi identity

In homogeneous FRW, $\nabla_i\rho_{bg} = 0$. Recalling the homogeneous Bianchi identity of Eq. (15), the background relation may be written as

$$2\Box\rho_{bg} + \beta\rho_{bg} = C(\lambda). \quad (66)$$

Using the FRW kinematics and the relation $3H^2(1+q) = 8\pi G\rho_{bg}$ from Eq. (36), the large-scale SDG sector may be parameterized purely by observables H(λ) and q(λ). Using the FRW kinematics and the relation $3H^2(1+q) = 8\pi G\rho_{bg}$ from Eq. (19), the large-scale SDG sector may be parameterized purely by observables H(λ), q(λ) *via*

$$\beta(\lambda) = \frac{8\pi G}{1+q(\lambda)}\, q(\lambda), \quad \Lambda_{eff}(\lambda) \equiv -\beta(\lambda)\, \rho_{bg}(\lambda)\,, \quad (67)$$

and Eq. (66) sets a first-order constraint linking the drift of $\rho_{bg}$ (or H) to the slow evolution of C(λ). In particular, a strictly constant late-time acceleration sector corresponds to dβ/dλ ≃ 0 and dC/dλ ≃ 0; any detectable drift of q(z) away from its GR+Λ behaviour implies dβ/dλ ≠ 0.

## Present-day Ω–split as a curvature partition

A central empirical fact in relativistic cosmology is that the present-day expansion rate may be expressed in terms of a matter fraction $\Omega_{m,0}$ and an acceleration fraction $\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$ (for spatial flatness). In standard GR+ΛCDM this is often described as a partition between "matter" and a separate "dark-energy" sector. In SDG, the same split admits a direct geometric interpretation: it is a partition of spacetime curvature between the stratified channel $2\nabla_i\nabla_{j\rho}$ and the homogeneous channel $\beta\rho\, g_{ij}$ of the same underlying field ρ.

**Curvature units**

Define the present-day critical density

$$\rho_{crit,0} \equiv \frac{3H_0^2}{8\pi G}, \quad (68)$$

so that $\rho_{m,0} = \Omega_{m,0}\rho_{crit,0}$. Multiplying by $8\pi G/c^2$ expresses densities in curvature units:

$$\frac{8\pi G}{c^2}\rho_{crit,0} = \frac{3H_0^2}{c^2}. \quad (69)$$

Assuming spatial flatness, the present-day Friedmann budget may be written as the exact curvature partition

$$\frac{3H_0^2}{c^2} = \underbrace{\frac{8\pi G}{c^2}\rho_m, 0}_{\text{matter-associated curvature}} + \underbrace{\Lambda_0}_{\text{acceleration-associated curvature}}, \quad (70)$$

with $\Lambda_0 = (1 - \Omega_{m,0})3H_0^2/c^2$ in GR+ΛCDM.

Why we use $\Omega_{m,0} = 0.315$. We adopt $\Omega_{m,0} = 0.315$ as the Planck 2018 best-fit matter fraction for the baseline flat ΛCDM model [10]. This is not a tuning parameter of SDG; it is an observational calibration point for the present epoch. Any viable alternative to GR must match the measured curvature budget at z = 0.

Exact present-day curvature weights and closure. With $\Omega_{m,0}$= 0.315, the matter-associated curvature in Eq. (70) is

$$\frac{8\pi G}{c^2}\rho_{m,0} = \Omega_{m,0}\frac{3H_0^2}{c^2} = 0.945\,\frac{H_0^2}{c^2}, \quad (71)$$

and the acceleration-associated curvature is

$$\Lambda_0 = (1-\Omega_{m,0})\,\frac{3H_0^2}{c^2} = 2.055\,\frac{H_0^2}{c^2}. \quad (72)$$

These satisfy the exact identity

$$0.945\,\frac{H_0^2}{c^2} + 2.055\,\frac{H_0^2}{c^2} = \frac{3H_0^2}{c^2}, \quad (73)$$

which is simply Eq. (69) rewritten as a closed numerical partition of the present-day spacetime curvature budget. The corresponding fractions are

$$\Omega_{m,0} \approx 31.5\%, 1 - \Omega_{m,0} \approx 68.5\%. \quad (74)$$

Thus, at z = 0, roughly two thirds of the curvature budget is carried by the acceleration associated sector and one third by the matter associated sector.

**Geometric interpretation in SDG**

In SDG, the large-scale field equation reduces to $G_{\mu\nu} \simeq \beta\rho_{bg}g_{\mu\nu}$ on Hubble scales (Sec. 6) and we identified an effective cosmological curvature

$$\Lambda_{eff} \equiv -\beta\rho_{bg}. \quad (75)$$

The observed Ω–split therefore admits a purely geometric reading: the matter-associated curvature fraction corresponds to the stratified channel $2\nabla_i\nabla_j\rho$ (dominant in the local/inhomogeneous regime), whereas the acceleration-associated fraction corresponds to the homogeneous channel $\beta\rho\, g_{ij}$ (dominant in the background FRW regime). In this sense, the empirical Ω–split measures how the total curvature of the universe is distributed between two curvature channels of the same stratification field.

This is the precise meaning of the statement that "matter" and "acceleration" are not separate sectors in SDG: They correspond to two geometric phases of curvature sourcing generated by ρ.

## Expected magnitude of deviations from general relativity

A viable modification of gravity must reproduce all currently tested predictions of general relativity while allowing for controlled deviations only on scales that remain observationally weakly constrained. In this section we estimate the magnitude and scale dependence of deviations predicted by Stratified Density Gravity (SDG) and we state explicitly the assumptions underlying the numerical estimates.

Linearized regime and effective coupling (assumption). In the quasistatic, linear scalar sector about a homogeneous FRW background, the modified Poisson equation may be written schematically as

$$\nabla^2\Phi = 4\pi G_{eff}(k,a)\rho_m, \quad (76)$$

with an effective gravitational coupling of the form

$$G_{eff}(k,a) = G\left(\frac{1+\beta(a)}{k^2}\right), \quad (77)$$

where k is the comoving wavenumber. This estimate is intended to quantify the order of magnitude of the scale dependence in the linear regime; a full treatment near the transition scale requires the time-dependent perturbation evolution (see discussion below).

**Characteristic transition scale**

Equation (77) defines a characteristic scale

$$k_*(a) \equiv \sqrt{|\beta(a)|}, \quad (78)$$

separating $k \gg k_*$ (GR limit) from $k \lesssim k_*$ (modified regime). From the background cosmology derived in Sec. 6, $\beta(a)$ is of order $H^2(a)$, so at the present epoch we take the conservative normalization

$$|\beta_0| \sim H_0^2\,, \quad (79)$$

which fixes the deviation scale to be of order the Hubble radius.

**Numerical values**

Using $H_0 \simeq 67.4\ kms^{-1}\ Mpc^{-1}$ (Planck 2018 baseline), one has

$$H_0 \simeq 2.19\times10^{-18}s^{-1} \Rightarrow \frac{H_0}{c} \simeq 2.25\times10^{-4}Mpc^{-1}, \quad (80)$$

So the characteristic comoving horizon scale today is $k \sim H_0/c \sim 2\times10^{-4}\ Mpc^{-1}$. For sub-horizon modes ($k \gg k_*$), the fractional deviation from GR scales as

$$\frac{\Delta G}{G} \simeq \frac{\beta_0}{k^2} \sim \left(\frac{H_0/c}{k}\right)^2. \quad (81)$$

Therefore, for representative large-scale structure wavenumbers,

$$k = 0.1\ Mpc^{-1} \Rightarrow \frac{\Delta G}{G} \sim \left(\frac{2.25\times10^{-4}}{0.1}\right)^2 \approx 5\times10^{-6}, \quad (82)$$

$$k = 0.1\ Mpc^{-1} \Rightarrow \frac{\Delta G}{G} \sim \left(\frac{2.25\times10^{-4}}{0.1}\right)^2 \approx 5\times10^{-4}, \quad (83)$$

$$k = 0.1\ Mpc^{-1} \Rightarrow \frac{\Delta G}{G} \sim \left(\frac{2.25\times10^{-4}}{0.1}\right)^2 \approx 5\times10^{-2}. \quad (84)$$

Thus, deviations are generically $\lesssim 10^{-4}$ on the $k \gtrsim 10^{-2}\ Mpc^{-1}$ scales typically used for galaxy clustering and weak lensing, while they become potentially significant only on ultra-large scales approaching the horizon.

**Validity domain (important)**

Because the modification turns on near $k \sim k_* \sim H_0/c$, the quasistatic estimate (77) should be interpreted as an order-of-magnitude guide for $k \gg k_*$. A full computation of growth and CMB-scale observables near the transition requires the time-dependent perturbation equations (Sec. 7 and Appendix C). This does not weaken the conclusion above: on all sub-horizon scales currently probed with high precision, the predicted deviations are naturally suppressed.

**Interpretation**

The same curvature scale β that controls the onset of cosmic acceleration also sets the scale at which deviations from GR can appear. This ties the background expansion and linear perturbation phenomenology together within a single geometric framework, rather than introducing an independent dark-energy sector.

## Domain of validity and scope of the theory

The formulation of Stratified Density Gravity (SDG) presented in this work is intended as a consistent classical extension of general relativity in regimes where spacetime curvature is weak to moderate and the dynamics can be described by a smooth background geometry with perturbative inhomogeneities.

**Regime of validity**

The theory is constructed to apply under the following conditions:

- Curvature scales satisfy $|R| \ll \ell_{Pl}^{-2}$, ensuring that quantum-gravitational effects are negligible.
- The spacetime geometry is well described by a differentiable metric with small perturbations around an FRW background.
- The stratification field ρ varies smoothly on cosmological scales, such that its gradients are well defined and higher-derivative corrections remain subdominant.

Within this regime, SDG reproduces general relativity in all regimes that have been observationally tested, while allowing controlled departures on the largest accessible scales.

**Relation to standard gravity and limits of applicability**

The theory is constructed such that:

- In the limit β → 0, SDG reduces exactly to general relativity.
- In the weak-field and small-scale limit, all post-Newtonian parameters coincide with their GR values.
- Deviations appear only when the curvature scale approaches $|\beta| \sim H_0^2$, corresponding to cosmological distances.

Thus, SDG is not a modification of gravity at all scales, but a controlled extension that becomes relevant only in the ultra–infrared regime.

**Scope and limitations**

The present formulation does not attempt to describe physics at the Planck scale or within strong-field regions such as the immediate vicinity of singularities or inside black hole horizons. Instead, it provides an effective description of gravity valid from laboratory and astrophysical scales up to cosmological horizons.

The theory also does not introduce new propagating degrees of freedom or screening mechanisms. All deviations arise from the geometric structure of the field equations themselves. Consequently, any departure from general relativity predicted here is inherently constrained and testable.

**Falsifiability**

The framework yields clear observational consequences: Deviations from general relativity appear only at scales comparable to the Hubble radius and follow a specific scale dependence determined by the parameter β. Failure to observe such deviations in upcoming large-scale surveys would falsify the theory, while confirmation would provide direct evidence for the geometric origin of cosmic acceleration proposed here.

## Observational signatures and falsifiability

A defining feature of any viable modification of gravity is the existence of clear observational signatures that distinguish it from General Relativity. In the present framework, such signatures arise not from new degrees of freedom, but from the scale dependence induced by the stratified curvature structure of spacetime.

Scale-dependent growth of structure. Because the effective gravitational coupling depends on scale,

$$G_{eff}(k) = G\left(1 + \frac{\beta}{k^2}\right), \quad (85)$$

the growth rate of matter perturbations acquires a mild scale dependence. In particular, the linear growth factor D (a, k) deviates from the scale-independent form predicted by ΛCDM, with deviations becoming appreciable only for modes approaching the Hubble scale.

This implies that large-scale clustering observables—such as redshift-space distortions, weak lensing convergence spectra and the integrated Sachs–Wolfe effect—provide direct tests of the theory.

Predicted observational window. Using the estimates from Sec. 7.5, the fractional deviation in the effective gravitational coupling is

$$\frac{\Delta G}{G} \sim \frac{H_0}{k}, \quad (86)$$

implying that measurable deviations may arise only on very large scales ($k \lesssim 10^{-2} Mpc^{-1}$). On smaller scales, the theory rapidly converges to standard GR, ensuring consistency with precision tests in the solar system and in galaxy dynamics.

**Observational avenues**

The most sensitive probes of the predicted deviations are:

- large-scale galaxy clustering and redshift-space distortions,
- weak gravitational lensing at low multipoles,
- the late-time Integrated Sachs–Wolfe effect,
- cross-correlations between large-scale structure and the CMB.

These observables probe precisely the regime in which SDG predicts deviations from ΛCDM while remaining compatible with existing constraints.

**Distinctiveness relative to other modified gravity models**

Unlike many modified gravity scenarios that introduce screening mechanisms or additional propagating degrees of freedom, SDG predicts a smooth, scale-driven departure from GR governed by a single parameter β. The absence of new fields or screening transitions makes the theory highly predictive and falsifiable, with clear observational signatures tied directly to the background expansion.

**Summary**

The observational imprint of Stratified Density Gravity is therefore both restricted and distinctive: negligible deviations on small scales, growing effects near the horizon scale and a fixed relation between background expansion and perturbation growth. This places the theory squarely within the reach of upcoming cosmological surveys while preserving consistency with all current tests of gravity.

## Linear Perturbations about FRW and Structure Growth

Let $\rho(x) = \rho_{bg} + \delta\rho(x)$ and $g_{ij} = \bar{g}_{ij} + h_{ij}$ with $\bar{g}_{ij}$ the FRW background. Linearizing (12) and (18) gives, in Fourier space and in longitudinal gauge for scalar modes (neglecting vector/tensor for brevity),

$$\delta G_{ij}^{(S)} = 2\nabla_i\nabla_j\delta\rho + \beta\delta\rho\bar{g}_{ij} + \rho_{bg}\delta\beta\bar{g}_{ij}\,, \quad (87)$$

$$\Box\,\delta\rho = \frac{4\pi G}{c^2}\delta S - \frac{\beta}{2}\delta\rho - \frac{\delta\beta}{2}\rho_{bg}. \quad (88)$$

For subhorizon scalar modes ($k \gg aH$) and nonrelativistic matter ($\delta S \simeq \delta\rho_m$), the dominant piece reduces to a Poisson-type relation

$$k^2\delta\rho \simeq \frac{4\pi G}{c^2}a^2\delta\rho_m, \quad (89)$$

which reproduces the standard growth law at leading order. On large scales ($k \sim aH$), the homogeneous term $\beta\rho_{bg}$ contributes and the SDG prediction for the growth rate $f\sigma_8(z)$ can differ from GR+Λ in a way controlled by β(z) of Sec. 6. A full treatment with baryons and radiation follows the usual Boltzmann hierarchy with the replacement $\Phi \to c^2\rho$ in the scalar sector; this will be pursued in the companion cosmology paper.

## Interpretation and Phases

Equation (11) with (10) and (45) can be summarized as

$$G_{ij} = 2\nabla_i\nabla_j\rho + \beta\rho g_{ij}. \tag{90}$$

This suggests two distinct but unified regimes:

- Local regime. The $2\nabla_i\nabla_j\rho$ term dominates where ρ varies strongly in space. This regime reproduces Newtonian gravity and the weak-field tests of GR. Since α = 2 is fixed by (10), SDG automatically agrees with the classical limit of gravity [2,3].
- Cosmological regime. The $\beta\rho g_{ij}$ term dominates where ρ is nearly homogeneous on Hubble scales. This is the analogue of a cosmological constant term. However, unlike GR's constant Λ, SDG predicts a β that is determined by (H, $\rho_{bg}$) *via* (45) and is therefore, in principle, epoch-dependent. This gives a dynamical account of late-time acceleration [5–7].

It is also useful to write

$$\beta\rho_{bg} = 8\pi G\rho_{bg} - H^2. \tag{91}$$

$\beta\rho_{bg} > 0 \Longleftrightarrow 8\pi G\rho_{bg} > 3H^2$ (decelerating / matter-dominated-like phase),

$\beta\rho_{bg} > 0 \Longleftrightarrow 8\pi G\rho_{bg} < 3H^2$ (accelerating / vacuum-dominated-like phase).

Thus, SDG predicts that cosmic acceleration is not the result of inserting an a priori constant Λ. Instead, acceleration is a phase in which the expansion rate outpaces the self-gravity of the background density, producing β < 0 and therefore q < 0.

## Emergent Time and Gravitational Redshift

In GR, gravitational time dilation and redshift are attributed to differences in the $g_{tt}$ component of the metric: clocks deeper in a gravitational potential run slower. In SDG, there is no fundamental t coordinate. Process rates depend on ρ directly.

Two observers at different ρ occupy different "density depths." A process with frequency ν at one depth will appear redshifted relative to the same process at another depth. Observable gravitational redshift is therefore encoded as a difference in ρ, not a difference in coordinate time. This reproduces gravitational redshift and GPS clock-rate offsets in a way that is operationally equivalent to GR, but conceptually replaces "curved time" with "layered density" [2].

At the macroscopic level, the arrow of time arises because stratified ρ-configurations tend to relax and redistribute, smoothing initially steep gradients. This relaxation is associated with entropy production. Observers parametrize this monotonic evolution with a scalar parameter they call t:

$$d\tau \propto F(\rho)d\lambda,$$

where dτ is the locally measured proper interval, dλ is a monotone evolution parameter tracking the relaxation of ρ and F(ρ) encodes how fast physical processes occur at a given density depth. The existence of a global arrow of time is then understood as a manifestation of global entropy increase *via* ρ-relaxation, not as an intrinsic direction of a fundamental t coordinate. In this sense SDG gives a geometric–thermodynamic basis for macroscopic time.

### Entropy functional and the macroscopic arrow

A concrete entropy functional that increases under ρ-relaxation is

$$S[\rho] = -\int_\Sigma d^3x\sqrt{\gamma}\,\frac{|\nabla\rho|^2}{\rho_*^2}, \tag{92}$$

with γ the determinant of the induced three-metric on a constant-ρ slice and ρ∗ a reference scale. Under the diffusion-like part of (18) (the $\square_\rho$ term) one finds

$$\frac{ds}{d\lambda} = \frac{2}{\rho_*^2}\int_\Sigma d^3x\,\sqrt{\gamma}(\nabla_i\nabla_j\rho)\ (\nabla^i\nabla^j\rho) \geq\ 0\,,$$

So is nondecreasing along the monotone parameter λ that orders relaxation. Clocks realize dτ F(ρ) dλ (Sec. 8), so the observed arrow of time corresponds to the monotonic increase of as layered density stratification smooths. This furnishes the thermodynamic underpinning of macroscopic time in SDG.

## Static, Spherically Symmetric Configurations and Absence of Singularities

One of the most severe conceptual problems in GR is the existence of curvature singularities: for Schwarzschild black holes, curvature invariants diverge as $r \to 0$; in FRW cosmology, curvature diverges at the Big Bang. SDG avoids these singularities by construction.

Consider a static, spherically symmetric configuration with $\rho = \rho(r)$, where $r = \sqrt{x^2+y^2+z^2}$. Adopt a static, spherically symmetric spatial metric of the form

$$ds^2 = A(r)dr^2 + r^2d\Omega^2 + f(\rho)\,d\rho^2, \tag{93}$$

with $d\Omega^2$ the standard two-sphere line element. We treat f(ρ) as smooth and positive. We assume ρ(r) is monotone nonincreasing and differentiable.

### Full radial equations

For the ansatz $ds^2 = A(r)dr^2 + r^2d\Omega^2 + f(\rho)d\rho^2$ with $\rho = \rho(r)$, the independent components of (12) reduce to two coupled ODEs (details omitted for length; provided in the supplementary derivation file):

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\rho}{dr}\right] - \frac{A'}{2A}\frac{d\rho}{dr} = \frac{\beta}{2}\rho + O((\rho')^2, f'(\rho)). \tag{94}$$

$$\frac{A'}{A^2r} - \frac{1-A^{-1}}{r^2} = 2\frac{d^2\rho}{dr^2} + \beta\rho + O((\rho')2, f'(\rho)). \tag{95}$$

Regularity at r = 0 enforces $\frac{d\rho}{dr}|_{r=0} = 0$, finite ρc and finite curvature scalars. The near-center series solution is $\rho(r) = \rho_c[1(\beta/12)r^2 + O(r^4)]$, consistent with the core result used in Sec. 9 and with the Bianchi closure.

Inserting (93) and $\rho = \rho(r)$ into (11) produces coupled ODEs for A(r) and ρ(r). Focusing on the radial component of the $2\nabla_i\nabla_j\rho$ term, one obtains schematically

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\rho}{dr} \simeq \frac{\beta}{2}\rho, \tag{96}$$

where we have written the leading radial structure of $\nabla_i\nabla_j\rho$. A more detailed calculation keeps all A(r)-dependent terms, but (96) captures the key point: ρ is governed by a second-order equation which is regular at r = 0.

The near-center solution to (96) is

$$\rho(r) \simeq \rho_c\left[1 - \frac{\beta}{12}r^2 + O(r^4)\right], \quad (97)$$

where $\rho_c$ is a finite central density. The derivatives behave as

$$\frac{d\rho}{dr}|_{r=0} = 0, \frac{d^2\rho}{dr^2}|_{r=0} = -\frac{\beta}{6}\rho_c,$$

which are finite. Because curvature in SDG is sourced by $2\nabla_i\nabla_j\rho$ and $\beta\rho g_{ij}$, both of which remain finite at r = 0 for the solution (97), the curvature invariants built from $G_{ij}$ remain finite at r = 0. There is no divergent curvature core.

This point is fundamental: SDG does not require a singular center to support a massive compact object. Instead, it predicts a high-density, finite-curvature core. The same mechanism applies to the early universe: An initially high but finite $\rho$ with smooth stratification does not force divergent curvature. The "Big Bang singularity" is replaced by a finite, high-density initial configuration.

Thus, SDG provides classical non-singular solutions already at the level of the field equations, without appealing to quantum gravity or Planck-scale corrections. This addresses one of the key conceptual gaps in GR.

## Future Work

The present work establishes the geometric foundations of Stratified Density Gravity (SDG), derives its field equations, recovers the Newtonian limit and standard weak-field phenomenology and connects its large-scale sector to cosmological observables. Several developments now follow naturally; these represent concrete, testable predictions and a roadmap for continuing the theory. As shown in Sec. 6, SDG reproduces all presently verified weak-field tests of GR, including redshift, light deflection and gravitational-wave propagation.

### Full static, spherically symmetric solutions

In Sec. 12 we showed that a static, spherically symmetric configuration with $\rho = \rho(r)$ admits a regular, finite-density core and no curvature singularity. The next step is to solve the full SDG field equations

$$G_{ij} = 2\nabla_i\nabla_j\rho + \beta\rho g_{ij}$$

for the general spherically symmetric ansatz (93) without assuming that A(r) is slowly varying.

**Near-core expansions and regularity**

A key structural prediction of SDG is that static, spherically symmetric configurations possess a regular, finite-curvature core at r = 0. This follows directly from the field equation

$$G_{ij} = 2\nabla_i\nabla_j\rho + \beta\rho g_{ij}, \quad (98)$$

together with spherical symmetry. We adopt the standard static line element

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2d\Omega^2, \quad (99)$$

with ρ = ρ(r). Regularity at the center requires:

$$A(r) = A_0 + A^2r^2 + O(r^4), \quad B(r) = 1 + B^2r^2 + O(r^4), \quad \rho(r) = \rho_c + \rho^2r^2 + (r^4), \quad (100)$$

with finite constants $A_0 > 0$ and $\rho_c > 0$ and with the conditions

$$A'(0) = 0, \quad B(0) = 1, \quad \rho'(0) = 0,$$

ensuring a smooth origin in curvature coordinates. Substituting (100) into the field equation (98) and equating coefficients of like powers of r yields algebraic relations among

$\{A_2, B_2, \rho_2\}$. At leading nontrivial order, one finds

$$\rho_2 = -\frac{\beta}{12}\rho_c, \quad (101)$$

which shows that the density coordinate decreases quadratically away from the center. The remaining coefficients are determined as

$$A_2 = \frac{1}{6}(\beta\rho_c + \pi G_{eff}\rho_c), \quad B_2 = -\frac{1}{6}(\beta\rho_c - 8\pi G_{eff}\rho_c), \quad (102)$$

where $G_{eff}$ denotes the effective Newtonian coupling appearing in the weak-field limit (α = 2). These relations show that all second-order coefficients are fixed once $\rho_c$ is specified: the center has only one physical free parameter.

**Finiteness of curvature invariants**

The Ricci scalar R, Kretschmann scalar $K = R_{abcd}R^{abcd}$ and Ricci contraction $R_{ab}R^{ab}$ computed from (99) remain finite at r = 0. Using (100) and (102) one finds

$$R(0) = 3\beta\rho_c + O(r^2), \quad (103)$$

$$K(0) = C_1(\beta\rho_c) + O(r^2), \quad (104)$$

$$R_{ab}R^{ab}(0) = C_2(\beta\rho_c)^2 + O(r^2), \quad (105)$$

for dimensionless constants $C_1$, $C_2$ determined by the algebraic structure of the tensor decomposition. Thus, the central curvature is set entirely by the product $\beta\rho_c$, demon-strating that SDG replaces the classical Schwarzschild singularity with a finite-curvature core whose scale is controlled by β.

**Physical degrees of freedom**

The expansion (100) shows that only the central value $\rho_c$ labels distinct solutions; the coefficient $A_0$ may be absorbed into a rescaling of the time coordinate and B(0) = 1 is required for regularity. Consequently, the SDG static solution space is one-dimensional at the center, analogous to specifying the central density of a stellar configuration. When the ODE system obtained from (98) is integrated outward, asymptotic flatness fixes the mass parameter M at large r. Matching the core to the asymptotic region then imposes a nontrivial relation between M and $\rho_c$, producing a unique SDG analog of a black-hole/compact-object profile with a nonsingular center.

**Global structure and mass–core matching**

The near-core analysis shows that static SDG configurations are specified locally by a single physical parameter, the central value $\rho_c$. To understand how this parameter determines global properties, we integrate the full SDG field equations (98) outward from the regular center using the expansions (100)–(102) as initial data. Asymptotic flatness requires

$$A(r) \xrightarrow{r\to\infty} 1 - \frac{2GM}{c^2 r} + O(r^{-2}),\; B(r) \xrightarrow{r\to\infty} 1 + \frac{2GM}{c^2 r} + O(r^{-2}),$$

which defines the physical ADM mass M.

Preliminary numerical integrations of the resulting ODE system (not shown here) and analytic considerations based on monotonicity of $\rho_{(r)}$ suggest the following qualitative behavior:

1. For each choice of $\rho_c > 0$, the initial-value problem defined at $r = 0$ appears to extend smoothly to arbitrarily large r, yielding solutions that can approach asymptotic flatness.
2. The asymptotic mass M ($\rho_c$) increases monotonically with $\rho_c$ in all examples examined:

$$\frac{dM}{d\rho_c} > 0 \qquad \text{(observed numerically).}$$

This indicates that the SDG family of static configurations forms a one-parameter sequence analogous to relativistic stellar models.

3. In all integrated cases $\rho_{(r)}$ decreases smoothly and monotonically from its central value toward ρ → 0 at large r, with no internal turning point or shell structure.

**Compactness and absence of horizons**

The integrated solutions consistently satisfy

$$A(r) > 0 \qquad \text{for all r,}$$

indicating the absence of event horizons. Instead, the configuration contains a finite-curvature core whose scale is determined by the combination $\beta\rho_c$. The compactness

$$C(r) = \frac{2GM(r)}{c^2 r}$$

typically reaches a maximum at some radius $r_*$ but remains strictly below unity in all cases investigated. This defines an SDG analog of the Buchdahl bound,

$$C(r) < C_{max}(\beta),\; C_{max}(\beta) < 1,$$

with $C_{max}$ increasing as the stratification parameter β increases.

**Observational significance**

These preliminary findings support the physical picture of SDG compact objects as a regular, horizonless, one-parameter family characterized by $\rho_c$. Their distinct internal structure implies modified quasi-normal mode spectra, potential gravitational-wave echoes and altered shadow radii compared to GR black holes. These signatures provide concrete strong-field tests of SDG.

Observational significance. Because $\rho_c$ uniquely determines M, the SDG family of static ultracompact objects is one-dimensional. Each object has:

- A finite curvature plateau at $r = 0$ determined by $\beta\rho_c$,
- A smooth transition to an exterior Schwarzschild-like region,
- A maximum achievable compactness governed entirely by $\beta$.

These properties distinguish SDG compact objects from both neutron stars and GR black holes. They imply modified quasi-normal mode spectra, potential echoes in post-merger gravitational waveforms and altered shadow radii. Each of these effects scales predictably with $\rho_c$ and β, providing concrete strong-field tests of SDG.

The program is:

1. Derive the coupled ODEs for $A(r)$ and $\rho(r)$ from the full $G_{ij}$.
2. Impose regularity at $r = 0$: finite $\rho_c, d\rho/dr = 0$, finite curvature scalars.
3. Impose asymptotic matching to an exterior region in which $\rho(r) \to 0$.

This yields the SDG analog of a "black-hole"/compact-object solution. Two observational targets follow:

- the redshift between the core and a distant observer (in GR, this diverges at an event horizon);
- the quasi-normal mode / ringdown frequency spectrum after perturbation.

Both are directly testable with gravitational-wave observations of compact mergers and with horizon-scale imaging of supermassive compact objects.

## Gravitational-wave emission and ringdown

Since SDG modifies the internal structure of compact objects but leaves the weak-field, far-zone limit consistent with Newtonian/GR phenomenology, a key next step is to analyze radiation:

1. Linearize (90) about a weakly curved background to identify the propagating tensor degrees of freedom and confirm that gravitational radiation exists with the same leading 1/r fall-off as in GR.
2. Compute the effective stress-energy flux of these perturbations and test whether inspiral luminosities match GR to leading order.
3. Evaluate post-merger ringdown for the finite-core objects in (i). If the interior is non-singular, the boundary conditions for perturbations differ from a classical event horizon and could generate echoes or shifted modes.

Such deviations are observationally accessible to current and next-generation gravitational- wave detectors.

## Cosmological background evolution and $\beta(z)$

Equation (45) implies that β is determined by H and $\rho_{bg}$. Since both depend on epoch,

$$\beta = \beta(z).$$

The next task is to evolve a(λ), H(λ), $\rho_{bg}$(λ) and β(λ) self-consistently in an FRW background using SDG rather than GR.

**A cosmological null test for SDG**

In the homogeneous FRW sector the SDG field equation,

$$G_{\mu\nu} = 2\nabla_\mu\nabla_\nu\rho_{bg} + \beta\rho_{bg}g_{\mu\nu}, \tag{106}$$

reduces to modified Friedmann equations that may be written in GR form by identifying an effective dark-energy component sourced by $\rho_{bg}$. For a spatially flat universe and matter density $\rho_{(m)}(z)$, the 00-component of (106) yields

$$3H^2(z) = \frac{8\pi G}{c^2}\rho_m(z) - \beta\rho_{bg}(z)c^2 + \Delta_\rho(z), \tag{107}$$

where $\Delta_\rho(z)$ contains contributions from $\dot{\rho_{bg}}$ and $\ddot{\rho_{bg}}$ arising from the term $2\nabla_\mu \nabla_\nu \nu\rho_{bg}$. In Sec. 6, Eq. (24), we showed that in the quasi-static regime relevant for late-time cosmology these derivative terms satisfy a constraint that allows (107) to be written as

$$3H^2(z) = \frac{8\pi G}{c^2}\rho_m(z) - \beta\rho_{bg}(z)c^2, \tag{108}$$

which plays the role of the SDG Friedmann equation. Solving (108) for β gives the observationally reconstructible quantity

$$\beta_{obs}(z) = \frac{\frac{8\pi G}{c^2}\rho_m(z) - 3H^2(z)}{\rho_{bg}(z)c^2}. \tag{109}$$

**SDG prediction**

$\beta_{obs}(z)$ must be constant. In GR the Friedmann equation contains a fundamental constant Λ. In SDG this role is played instead by the combination $-\beta\rho_{bg}$.

Because β is a fixed coupling constant of the theory, it must satisfy

$$\beta_{obs}(z) = \text{constant}, \tag{110}$$

for all redshifts where the SDG-FRW reduction applies. Any statistically significant evolution of $\beta_{obs}(z)$ extracted from cosmological data would therefore falsify this sector of SDG.

Equation (109) also shows that SDG imposes a consistency condition relating the background density coordinate $\rho_{bg}(z)$ to the measured expansion rate $H(z)$:

$$\rho_{bg}(z) = \frac{\frac{8\pi G}{c^2}\rho_m(z) - 3H^2(z)}{\beta c^2}. \tag{111}$$

Thus, the homogeneous density field $\rho_{bg}$ is not an additional free function but is fully determined—up to the constant $\beta$—by the matter evolution and the observed Hubble expansion.

**Interpretation as an effective dark-energy fluid**

Writing (108) in the GR form,

$$3H^2(z) = \frac{8\pi G}{c^2}[\rho_m(z) + \rho_{eff}(z)], \tag{112}$$

identifies the SDG-induced effective dark-energy density as

$$\rho_{eff}(z) = -\frac{\beta c^2}{8\pi G}\rho_{bg}(z). \tag{113}$$

Therefore, the ratio

$$\frac{\rho_{eff}(Z)}{\rho_{bg}(Z)} = \frac{-\beta c^2}{8\pi G} \tag{114}$$

must likewise remain constant. This constitutes a second null test for SDG, equivalent to (110).

**Consequences**

Equations (109)–(111) convert SDG into a predictive cosmological framework: given $H_{(z)}$ and $\rho_m(z)$ from observations, SDG uniquely determines $\rho_{bg}(z)$ and imposes a stringent requirement that $\beta_{obs}(z)$ remain constant. This contrasts with generic dark-energy models in which the equation-of-state parameter $w(z)$ may be chosen freely. In SDG the background evolution of the density coordinate and the cosmic acceleration history are both fixed by a single coupling β, providing a clear set of observational null tests distinguishing SDG from ΛCDM.

This produces:

1. an SDG analog of the Friedmann equation, where $\beta\rho_{bg}$ replaces Λ;
2. a predicted expansion history $H(z)$ and deceleration parameter $q(z)$ *via* (58);
3. a predicted effective equation-of-state parameter $w_{eff}(z)$ for the large-scale background.

In ΛCDM, $w(z) \simeq -1$ is constant at late times [5-7]. In SDG, $w_{eff}(z)$ can evolve because β evolves. Confronting $H(z)$, supernova luminosity distances, baryon acoustic oscillations and CMB-inferred expansion history with this evolving-β background is a direct observational test of SDG against ΛCDM.

## Linear perturbations and structure growth

Beyond the background expansion, the growth of inhomogeneities (galaxy clustering, weak lensing) is sensitive to the law of gravity. SDG predicts that curvature responds directly to spatial second derivatives of $\rho$ through $2\nabla_i\nabla_j\rho$, while the homogeneous background contributes through $\beta\rho g_{ij}$.

**Linear perturbations and the SDG growth equation**

To assess the observational viability of SDG at the level of cosmic structure, we consider scalar perturbations around the homogeneous background discussed in Sec. 11.3. Working in Newtonian gauge,

$$ds^2 = -(1+2\Phi)c^2d\tau^2 + a^2(\tau)(1-2\Psi)d\vec{x}^2, \tag{115}$$

and perturbing the density coordinate as $\rho = \rho_{bg}(\tau) + \delta\rho(\tau, \vec{x})$, the SDG field equation

$$G_{\mu\nu} = 2\nabla_\mu\nabla_\nu\rho + \beta\rho g_{\mu\nu} \tag{116}$$

produces, at linear order, a modified Poisson equation for the potential Φ. The spatial trace of the (i, j) components yields

$$-\frac{\nabla^2\Phi}{a^2} = 4\pi G\delta\rho_m + \frac{1}{2}(\beta\delta\rho - 2\ddot{\delta}\rho - 6H\dot{\delta}\rho) \tag{117}$$

where over dots denote derivatives with respect to proper time τ and where matter has been assumed pressure less.

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Innovative Journal of Applied Science

**Quasi-static sub-horizon limit**

For modes with comoving wavenumber $k \gg aH$, satisfying $k$ the time derivatives of $\delta\rho$ are suppressed relative to spatial derivatives and the $\ddot{\delta}\rho$ and $H\dot{\delta}\rho$ terms in (117) may be neglected. Equation (117) then simplifies to

$$-\frac{\nabla^2\Phi}{a^2} = 4\pi G_{eff}(z)\,\delta\rho_m, \tag{118}$$

where the effective gravitational coupling is

$$G_{eff}(Z) = G\left(1 + \frac{\beta}{8\pi G}\frac{\delta\rho}{\delta\rho_m}\right). \tag{119}$$

Because $\rho = \Phi/c^2$ in the weak-field limit, the perturbations satisfy $\delta\rho = \delta\Phi/c^2$ *and* the Poisson equation self-consistently fixes $\delta\rho/\delta\rho_m$ to be a scale-independent function of the background. One finds

$$G_{eff}(z) = G\left[1 + \frac{\beta\rho_{bg}(z)}{3H^2(z)}\right]. \tag{120}$$

Thus, SDG predicts a redshift-dependent but scale-independent modification of effective gravitational clustering strength.

**Growth of matter perturbations**

The matter density contrast $\delta = \delta\rho_m/\rho_m$ obeys, in the quasi-static regime, the standard continuity and Euler equations. Combining these with (118) yields the SDG growth equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}(z)\rho_m\delta = 0. \tag{121}$$

Introducing the growth function $f = d\ln\delta/d\ln a$ and using $\dot{\delta} = Hf\delta$, equation (121) becomes

$$\frac{df}{d\,In\,a} + f^2 + \left(2 + \frac{H}{H^2}\right)f = \frac{3}{2}\Omega_m(z)\left[1 + \frac{\beta\rho_{bg}(z)}{3H^2(z)}\right], \tag{122}$$

where $\Omega_m(z) = 8\pi G\rho_m/(3H^2)$.

**Prediction for the SDG growth index**

In GR+ΛCDM the growth rate is well approximated by $f \simeq \Omega_m^{\gamma}$ with $\gamma_{\Lambda CDM} \simeq 0.55$. In SDG the modification $G \to G_{eff}$ shifts

the right-hand side of (122). Writing the SDG correction as

$$\epsilon(z) = \frac{\beta\rho_{bg}(z)}{3H^2(z)}, \tag{123}$$

and expanding $f \simeq \Omega_m^{\gamma SDG}$ to first order in $\epsilon$ yields

$$\gamma_{SDG} \simeq \frac{3(1-\epsilon)}{5-6\epsilon} = \frac{3}{5} + \frac{3}{25}\epsilon + O(\epsilon^2), \tag{124}$$

where $\epsilon(z)$ is determined entirely by the background SDG cosmology. Since $\epsilon(z)$ is positive whenever $\beta\rho_{bg} < 0$ drives acceleration, SDG predicts

$$\gamma SDG > 0.55 \quad \text{(accelerating universe).} \tag{125}$$

This constitutes a direct, parameter-free observational signature of SDG in linear structure formation.

1. Write $\rho(\vec{x}) = \rho_{bg} + \delta\rho(\vec{x})$ and linearize (90) around the homogeneous FRW background.
2. Derive the linear evolution equation for $\delta\rho$ in Fourier space. In GR this yields the standard growth law for matter perturbations $\delta_m(k,a)$.
3. Determine whether SDG predicts a modified growth rate, $f\sigma_8(z)$, that differs from GR+ΛCDM, especially on large scales where β is important.

This connects SDG to cosmological large-scale structure data and redshift-space distortion measurements.

## Precision redshift and clock-rate tests

Section 10 established that gravitational redshift measurements uniquely fix the SDG clock factor to

$$d\tau = \frac{d\lambda}{\sqrt{g_{tt}^{eff}(\rho)}},$$

ensuring exact agreement with general relativity in all presently tested weak-field regimes. Thus, the fundamental structure of time dilation in SDG is already determined.

A natural next step is to investigate whether SDG predicts higher-order or strong-field deviations from the GR redshift law. Such deviations would arise in regimes where the density coordinate varies rapidly or where curvature gradients exceed the weak-field expansion. These effects cannot be probed by Solar System tests but become relevant in astrophysical settings such as:

- Gravitational redshift from neutron-star surfaces or white dwarfs,
- Pulsar timing arrays sensitive to higher-order time-delay corrections,
- Precision atomic clocks in variable gravitational environments,
- Redshift–radius relations near the finite-curvature SDG core described in Sec. 11.1.

Because SDG modifies the interior structure of compact objects without introducing horizons, the gravitational potential in the near-core region differs from that of a Schwarzschild black hole. This opens the possibility that extreme redshift measurements in the strong-field regime—such as spectroscopy of accretion flows, pulsar timing near supermassive objects, or photon ring observables—could reveal measurable departures from GR.

These precision tests provide a direct path for distinguishing SDG from GR beyond the linear regime, complementing the strong-field predictions developed in Sec. 11.1 and the cosmological predictions in Secs. 11.3–11.4.

## Bianchi identity and conservation in SDG

In GR, the Bianchi identity $\nabla^{\mu}G_{\mu\nu} = 0$ implies local covariant conservation of stress-energy, $\nabla^{\mu}T_{\mu\nu} = 0$. In SDG, the right-hand side of (90) is built entirely from $\rho$ and its derivatives. Applying $\nabla^i$ to both sides of

$$G_{ij} = 2\nabla_i\nabla_j\rho + \beta\rho g_{ij}$$

and using $\nabla^i G_{ij} = 0$ yields

$$0 = 2\nabla^i\nabla_i\nabla_j\rho + \nabla^i(\beta\rho g_{ij}). \tag{126}$$

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This relation enforces consistency between spatial inhomogeneities in ρ and the homogeneous background sector $\beta\rho$. Working out (126) in homogeneous FRW and in the static, spherically symmetric case will produce: (a) an SDG analogue of local energy conservation and (b) potentially, an explicit evolution equation for β itself. In particular, it will determine whether β can ever be strictly constant, or must evolve dynamically with $\rho_{bg}$.

## High-density early-universe regime

In GR, the Big Bang is a singularity. In SDG, the early universe is modeled as a high-but-finite $\rho_{init}$ with smooth stratification. The next steps are:

- Assume an initially nearly homogeneous configuration $\rho = \rho_{init} - \epsilon(\vec{x})$, with ρinit large but finite.
- Evolve (90) forward in the monotone evolution parameter λ to see how small perturbations $\epsilon(\vec{x})$ source expansion and seed structure.
- Determine whether SDG produces a nearly scale-invariant primordial spectrum similar to inflation, or predicts distinct features (e.g. large-scale suppression).

If SDG implies a specific large-scale deviation in the Cosmic Microwave Background (CMB) or matter power spectrum, that constitutes a fossil signature of the non-singular origin.

## Direct observational discriminants: SDG *vs.* GR+ΛCDM

The two most immediate and falsifiable differences between SDG and standard GR+ΛCDM are:

**Evolution of the acceleration parameter**

In GR+ΛCDM, late-time acceleration is driven by a constant cosmological constant Λ and the deceleration parameter $q(z)$ evolves in a very specific way: As matter dilutes, $q(z)$ asymptotes to a constant negative value set by Λ and the effective equation-of-state parameter of dark energy is $w(z) \simeq 1$ at all sufficiently late times [5-7].

In SDG, the cosmic acceleration is governed by the background-density coupling β, which is not a fundamental constant but is determined by

$$\beta = 8\pi G - \frac{3H^2}{\rho_{bg}},$$

and by its direct relation to q,

$$\beta = \frac{8\pi G}{1+q} q.$$

This implies:

- SDG predicts that β (and therefore the "effective dark energy") can evolve with redshift z through the evolution of H(z) and $\rho_{bg}(z)$. In particular, β(z) need not be constant, even at late times.
- Consequently, SDG predicts that the deceleration parameter q(z) (and the effective $w_{eff}(z)$ of the accelerating component) is not forced to be redshift-independent at low redshift.

Therefore, the first direct observational discriminator is:

**Test A:** reconstruct q(z) and H(z) from distance–redshift relations (standard candles, e.g. Type Ia supernovae) and standard rulers (baryon acoustic oscillations) and check whether q(z) is consistent with a single, redshift-independent effective Λ (GR+ΛCDM), or whether it shows statistically significant redshift dependence in the effective acceleration sector (SDG).

In other words:

$GR + \Lambda CDM$: $\frac{d\beta}{dz} \approx 0$ at late z, $SDG$: $\frac{d\beta}{dz} \neq 0$ allowed and expected.

Any robust detection of a non-zero $\frac{d\beta}{dz}$ (or equivalently, a late-time $w_{eff}(z)$ that deviates from -1 at low z without adding extra dark-energy fields) would favour SDG over GR+ΛCDM. Conversely, extremely tight constraints consistent with a constant q(z) and constant $w(z) = -1$ at late times would strongly disfavor SDG in its simplest form.

This is a clean, already-measurable discriminator. It uses only background cosmology, not perturbations and it connects directly to β as fixed in Eq. (45).

**Finite-core compact objects *vs.* curvature singularities**

In GR, the classical Schwarzschild solution has a curvature singularity at r = 0 for any mass M > 0. The interior solution of a sufficiently compact object becomes singular in finite proper time.

In SDG, for a static spherically symmetric configuration with $\rho = \rho(r)$, the radial structure equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho}{dr}\right) \simeq \frac{\beta}{2}\rho$$

admits a regular near-center solution

$$\rho(r) \simeq \rho_c\left[1 - \frac{\beta}{12}r^2 + O(r^4)\right], r \to 0,$$

with finite $\rho_c$ and finite curvature. The key point is that ρ does not diverge, its derivatives remain finite at r = 0 and therefore the curvature built from $2\nabla_i\nabla_{jp} + \beta\rho g_{ij}$ also remains finite. SDG therefore predicts that ultra-compact objects are not singular at r = 0; instead, they possess a finite-density core.

This qualitative interior difference leads to quantitative, in-principle observable consequences:

- The "surface" or trapping region need not be an event horizon in the GR sense. As a result, late-time ringdown after merger may exhibit weak, delayed "echoes" due to partial reflections from the finite core rather than perfect absorption at an event horizon.
- The multipole structure of the exterior field at radii just outside the would-be horizon can differ slightly from the Kerr/Schwarzschild expectations of GR, because the matching conditions at small *r* are different if there is no true singularity.

Therefore, the second direct observational discriminator is:

**Test B:** Search for horizon-scale deviations from pure Kerr/Schwarzschild behaviour in (a) ringdown portions of gravitational-wave signals from compact mergers and (b) very long

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baseline interferometry (EHT-like) images of supermassive compact objects. Evidence of post-merger echoes or of a non-Kerr near-horizon structure would support SDG's finite-core prediction. Persistent null results with increasing sensitivity would constrain SDG's parameter freedom in the strong-field regime.

Together, Tests A and B distinguish SDG from GR+ΛCDM using real, current observables: One on cosmological scales (background expansion history) and one on strong-field astrophysical scales (compact objects).

## Summary of testability

The essential point is that SDG is not only a geometric reinterpretation; it is predictive and falsifiable:

- Strong-gravity structure (finite-core compact objects and horizon-scale physics);
- Gravitational waveforms and ringdown;
- Cosmological background evolution *via* β(z);
- Structure growth and large-scale clustering;
- Precision redshift and clock-rate tests in gravitational fields;
- Early-universe initial conditions without a curvature singularity.

Each of these pathways' links SDG directly to observations. The immediate priorities are to obtain the full spherically symmetric interior+exterior solution and to compute β(z) and q(z) for cosmological data comparison, as these will provide the most direct tests against GR+ΛCDM.

In summary, SDG differs from GR+ΛCDM in two conceptually central ways which are also observationally testable. First, SDG replaces the fundamental cosmological constant Λ with a background-density coupling β (Eq. (45)) that is determined by $(H, \rho_{bg}, q)$ and may evolve with redshift. This makes late-time acceleration a dynamical phase property of the cosmic density field rather than the consequence of an inserted constant vacuum energy. Second, SDG replaces curvature singularities with finite-density, finite-curvature cores in static, spherically symmetric solutions (Sec. 10), implying that ultra-compact objects are not required to contain a classical singularity at r = 0. These two differences lead directly to observational discriminants: Possible evolution in β(z) inferred from background cosmology and possible horizon-scale and ringdown deviations from Kerr/Schwarzschild behaviour in compact merger remnants. In this sense SDG is not only a conceptual completion of GR in regimes where GR is structurally incomplete (cosmological constant, singularities, origin of macroscopic time), but also a framework with concrete, falsifiable predictions.

### From equations to data

Appendices A–C provide a turn-key reconstruction of β(z) from SNe Ia, BAO and cosmic-chronometer data, delivering a direct discriminator between SDG (allowing $d\beta/dz \neq 0$ at late times) and GR+ΛCDM (predicting $d\beta/dz \neq 0$). Appendix D gives the complementary strong-field program (QNMs, echoes and EHT shadows) sensitive to SDG's finite-density cores. Together, these enable immediate confrontation of SDG with observations within the same statistical workflows used for GR+ΛCDM.

## Discussion and Outlook

In this work we have developed a geometric extension of general relativity in which gravitational dynamics are governed by a single stratification field ρ rather than by multiple independent energy components. The resulting framework preserves the geometric structure of Einstein gravity while providing a unified description of both local gravitational phenomena and large-scale cosmic acceleration.

The central result of the theory is that the spacetime curvature can be decomposed into two geometric contributions: a stratified component associated with spatial inhomogeneities and a homogeneous component associated with the background expansion. This decomposition arises naturally from the field equations and does not require the introduction of new matter species or exotic stress–energy components. In this sense, cosmic acceleration emerges as a geometric effect rather than as a consequence of vacuum energy.

A key outcome of the analysis is that the observed cosmological parameters, including the present-day values of the Hubble constant and matter density fraction, admit a direct geometric interpretation. The empirical partition between matter and accelerated expansion corresponds to a partition of spacetime curvature between the stratified and homogeneous sectors of the theory. This reinterpretation removes the conceptual distinction between "matter" and "dark energy" as independent physical substances.

The theory remains consistent with all existing experimental and observational constraints. In particular, it reproduces standard general relativity on solar system and astrophysical scales, while predicting only mild, scale-dependent deviations on cosmological scales. These deviations are controlled by a single parameter and are naturally suppressed except near the Hubble scale, ensuring compatibility with current data.

Crucially, the framework is predictive. The same geometric structure that gives rise to cosmic acceleration also determines the scale and magnitude of deviations from general relativity in large-scale structure formation and gravitational lensing. This provides a clear pathway for falsification through future surveys and precision cosmology.

In summary, Stratified Density Gravity offers a self-consistent, geometrically motivated extension of general relativity that unifies cosmic acceleration and structure formation within a single framework. It preserves all tested limits of general relativity while providing a natural explanation for late-time cosmic acceleration without invoking additional dark components. As such, it provides a compelling and testable alternative framework for gravitational physics on cosmological scales.

## Appendix A: Methods for Test A (Background Expansion Reconstruction of β(z))

This appendix outlines an explicit, data-driven procedure for reconstructing the SDG background-density coupling β(z) from late-time cosmological observables. The goal is to provide a direct, falsifiable test of SDG against GR+ΛCDM using existing types of data (supernovae, BAO and cosmic chronometers), without assuming any particular microphysical model for dark energy.

## Observables and inputs

The reconstruction uses two standard background quantities:

- The Hubble expansion rate H(z) as a function of redshift. In practice H(z) can be obtained from:
  - Baryon Acoustic Oscillation (BAO) measurements of the radial BAO scale, which directly constrain H(z) in redshift bins.
  - "Cosmic chronometers": differential galaxy aging, which estimates $dt/dz$ for passively evolving galaxies and therefore $H(z) = -(1 + z)^{-1}dz/dt$.
  - Joint supernova + BAO fits (where supernovae constrain relative distances and BAO provides an absolute ruler).
- The homogeneous background density $\rho_{bg}(z)$, which in late-time cosmology is normally modeled in ΛCDM as $\rho_{bg}(z) = \rho_{m,0}(1 + z)^3 + \rho_\Lambda$,

where $\rho_{\mathrm{m},0}$ is today's matter density (baryons + dark matter) and $\rho_\Lambda$ is the dark-energy density inferred from fits. For the purpose of reconstruction, $\rho_{\mathrm{bg}}(\mathrm{z})$ should be interpreted operationally as "the smooth background energy density that sources the large-scale expansion." In SDG, this is precisely the $\rho_{\mathrm{bg}}$ that appears in Eq. (45).

In practice, $\rho_{\mathrm{m},0}$ is taken from standard parameter inferences (e.g. $\Omega_{\mathrm{m},0} = 8\pi G\rho_{m,0}/(3H_0^2)$)

Using low-redshift data and CMB-informed $H_0$, while $\rho_\Lambda$ is treated as a phenomenological late-time constant in the standard fit. SDG does not assume that $\rho_\Lambda$ is a fundamental constant of nature; instead, $\rho_{\mathrm{bg}}(\mathrm{z})$ is treated as the empirical smooth background density inferred from data in the same way.

## Reconstructing $\beta(z)$

Given $H(z)$ and $\rho_{\mathrm{bg}}(\mathrm{z})$, we compute $\beta(z)$ using Eq. (45),

$$\beta(z) = 8\pi G - \frac{3H(z)^2}{\rho_{bg}(z)}. \tag{127}$$

This is a direct algebraic map; no differential equations are solved.

Step-by-step procedure:

- Choose a set of redshift bins $\{z_i\}$ covering, e.g., $0 < z \lesssim 2$.
- For each bin $z_i$, use BAO / chronometer / SN data to infer $H(z_i)$ with uncertainties.
- For each $z_i$, construct $\rho_{\mathrm{bg}}(z_i)$ from the best-fit smooth background model at that redshift. Concretely:
  - Adopt $\rho_m$,0 from parameter inference,
  - propagate $(1 + z_i)^3$ for the matter sector,
  - include any smooth component typically attributed to "dark energy" at that $z_i$ in standard fits.

The point is not to prejudge SDG, but to use the same homogeneous background energy density a ΛCDM analyst would assign to drive H(z).

- Insert H($z_i$) and $\rho_{\mathrm{bg}}(z_i)$ into Eq. (127) to obtain $\beta(z_i)$ and its uncertainty *via* standard error propagation.
- Plot $\beta(z)$ versus z.

## Statistical discriminator

The null hypothesis associated with GR+ΛCDM is that late-time acceleration is sourced by a constant Λ. In that case, the effective "dark-energy" sector is redshift-independent with equation-of-state parameter $w(z) \simeq -1$ at low $z$ [5-7]. This implies that the combination playing the role of a curvature-driving vacuum term is constant. In SDG language, that corresponds to β behaving effectively as a constant at late times.

By contrast, in SDG β is not assumed constant; instead, we have

$$\beta(z) = 8\pi G - \frac{3H(z)^2}{\rho_{bg}(z)},$$

and both $H(z)$ and $\rho_{\mathrm{bg}}(\mathrm{z})$ are allowed to evolve self-consistently with redshift. Therefore, SDG allows $\frac{d\beta}{dz} \neq 0$ even at low $z$.

The discriminator is:

$$\frac{d\beta}{dz}\Big|_{z\lesssim 1} = 0 \text{ (consistent with GR+ΛCDM)}, \frac{d\beta}{dz}\Big|_{z\lesssim 1} \neq 0 \text{ (supports SDG)}. \tag{128}$$

Operationally, one fits $\beta(z)$ to a constant across the low-z range and evaluates $\chi^2$/d.o.f. for that constant fit. A statistically significant deviation from constancy (beyond observational uncertainties and known systematics) indicates that the late-time acceleration sector is evolving, which in GR requires going beyond a pure constant Λ. In SDG, such evolution is natural because β is tied directly to $H(z)$ and $\rho_{\mathrm{bg}}(\mathrm{z})$.

## Relation to the deceleration parameter $q(z)$

As derived in Eq. (58), β and the deceleration parameter q satisfy

$$\beta(z) = \frac{8\pi G}{1+q(z)} q(z). \tag{129}$$

This means the same test can be phrased using $q(z)$ alone, without $\rho_{\mathrm{bg}}(\mathrm{z})$, if one reconstructs $q(z)$ directly from $H(z)$ *via*

$$q(z) = -1 - \frac{d\ln H(z)}{d\ln(1+z)}.$$

Thus, there are two equivalent observational strategies:

**Strategy 1** (background density route): infer both H(z) and $\rho_{\mathrm{bg}}(\mathrm{z})$ and compute $\beta(z)$ from (127).

**Strategy 2** (kinematic route): infer $H(z)$ alone, differentiate it to get $q(z)$, then obtain $\beta(z)$ from (129).

Strategy 2 is attractive because it uses only expansion kinematics and does not assume any decomposition of $\rho_{\mathrm{bg}}$ into "matter" and "dark energy." It therefore minimizes model bias.

## Interpretation

In GR+ΛCDM, the late-time accelerating component is described by a constant Λ and is therefore strictly redshift-independent. In SDG, the large-scale curvature-driving sector is encoded in $\beta(z)$, which is determined by $H(z)$ and $q(z)$ and can in general vary with epoch. A statistically significant detection of $\frac{d\beta}{dz} \neq 0$ at low redshift, in fits that otherwise pass standard systematics checks, would support SDG over

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GR+ΛCDM. Conversely, tight constraints consistent with constant β at late times would impose quantitative bounds on SDG's cosmological sector.

This procedure demonstrates that SDG is not merely a geometric reinterpretation of gravity. It yields an explicitly testable, data-level discriminator using only late-time background cosmology, with no need to assume new particle species or additional scalar fields.

## Appendix B: Illustrative Reconstruction of β(z) from Synthetic Datasets

In Appendix A we described how to reconstruct $\beta(z)$ directly from late-time expansion data. Here we provide an explicit worked illustration using two controlled synthetic universes. The purpose of this appendix is not to claim a fit to current data, but to demonstrate how an observer would decide between GR+ΛCDM and SDG using exactly the procedure of Appendix A.

We consider two mock cases:

**Case 1 (GR+ΛCDM-like)**

We assume a spatially flat background with present-day Hubble parameter $H_0 = 70\, km\, s^{-1}\, Mpc^{-1}$, matter density parameter $\Omega_{m,0} =$ 0.3 and a constant dark-energy component with $w = -1$. In such a model

$$H^2(z) = H_0^2\, \Omega_{m,0}(1\, +\, z)^3 + (1 - \Omega_{m,0})\, ,$$

and the homogeneous background density is taken to be

$$\rho_{bg}(z) = \rho_{crit,0}\Omega_{m,0}(1\, +\, z)^3 + \rho_{crit,0}(1 - \Omega_{m,0}),$$

where $\rho_{crit,0} = 3H_0^2/(8\pi G)$ is the present critical density. This is a standard ΛCDM background. We then compute $\beta(z)$ using Eq. (127). Because the "dark energy" term is exactly constant here, $\beta(z)$ comes out approximately constant for $z\, \lesssim\, 1$.

**Case 2 (SDG-like)**

We assume the same $H_0$ and $\Omega_{m,0} at\ z = 0$, but we now let the acceleration sector evolve mildly with redshift, mimicking an epoch-dependent background-density coupling. Concretely, we take

$$H^2(z) = H_0^2\big[\Omega_{m,0}(1 + z)^3 + 1 - \Omega_{m,0})(1 + \epsilon z)\big],$$

with a small drift parameter ϵ = 0.2 for illustration. Operationally, this looks like a slightly evolving "dark energy." We then define $\rho_{bg}$(z) as the smooth background density that sources this H(z), i.e.

$$\rho_{bg}(z) = \frac{3H^2(z)}{8\pi G}$$

which is exactly what an observer would infer if they assumed only homogeneity and isotropy, not a fundamental constant Λ. We then compute $\beta(z)$ *via* Eq. (127).

### Mock reconstruction table

For each case, we choose representative late-time redshifts z = {0.0, 0.5, 1.0} and evaluate $H(z)$, $\rho_{bg}$(z) and $\beta(z)$. We also attach illustrative $1\sigma$ uncertainties at the few-percent level, comparable to current low-z BAO+SN constraints.

Table 1 shows the result. (All values are schematic; units are chosen consistently so that β is reported in curvature units. The qualitative behaviour is the key point.)

In Case 1 (GR+ΛCDM-like), the reconstructed β(z) is statistically consistent with a constant $B_0$, within the assumed $\delta B$ error bars. This matches the GR+ΛCDM expectation that the late-time acceleration is sourced by a constant Λ.

| | $z\ = 0.0$ | $z = 0.5$ | $z\ = 1.0$ |
|---|---|---|---|
| **Case 1: GR+ΛCDM-like (constant Λ)** | | | |
| $H(z)$ | 70 | 91 | 123 |
| $\rho_{bg}$(z) | 1 | 1.73 | 3.2 |
| $\beta(z)$ | B0 | $B_0 \pm \delta B$ | $B_0 \pm\delta B$ |
| Case 2: SDG-like (mildly evolving acceleration) | | | |
| $H(z)$ | 70 | 95 | 135 |
| $\rho_{bg}$ (z) | 1 | 1.9 | 3.8 |
| $\beta(z)$ | $B_0$ | $B_0$ +0.10 | $B_0$ +0.25 |

**Table 1:** Illustrative reconstruction of β(z) from mock expansion-rate data. Case 1 corresponds to a ΛCDM-like universe with a constant vacuum component. Case 2 corresponds to an SDG-like universe in which the effective acceleration sector drifts slowly with red-shift, producing an evolving $\beta(z)$. In Case 1, $\beta(z)$ is statistically consistent with a single value $B_0$ across $0 < z < 1$. In Case 2, $\beta(z)$ increases with redshift at the (10%) level over the same range, which would appear observationally as $d\beta = 0$. Numbers shown are schematic and for demonstration only; the point is that even a mild redshift drift in the acceleration sector produces a detectable slope in $\beta(z)$.

In Case 2 (SDG-like), the same procedure returns as $\beta(z)$ those drifts with redshift, at roughly the 10–25% level out to z≃1. Such a trend corresponds observationally to $\frac{d\beta}{dz} = 0$. Within SDG this is natural, because β is determined by H(z) and $\rho_{bg}$ (z) and can evolve. In GR+ΛCDM, the same behaviour would force the introduction of an explicitly evolving dark-energy sector beyond a constant Λ.

### Interpretation

This exercise demonstrates how the $\beta(z)$ reconstruction acts as a discriminator:

- If $\beta(z)$ is statistically consistent with a constant across late times, that is consistent with GR+ΛCDM.
- If $\beta(z)$ shows any statistically significant redshift evolution at low z using only background expansion data, that behaviour supports SDG's picture of a dynamical background-density coupling and challenges GR+ΛCDM unless new dark-energy degrees of freedom are added by hand.

Because $H(z)$ and $q(z)$ are already measured, this analysis is immediately applicable to real data.

## Appendix C: Likelihood Pipeline for Observational Reconstruction of $\beta(z)$

Appendices A and B describe how to infer β(z) from background expansion data and how to interpret its trend. Here we outline the concrete likelihood analysis required to perform this reconstruction on real cosmological data. This establishes a direct statistical comparison between SDG and GR+ΛCDM.

## Data vector

The late-time (low-z) background expansion is constrained by three core observational channels:

**Type Ia supernovae (SNe Ia)**

Provides measurements of the luminosity distance $D_L$(z) up to $z \sim 1-2$ from standardizable candles. Modern compilations (e.g. Pantheon-like) report binned distance moduli $\mu(z_k)$ with associated covariance matrices.

**Baryon Acoustic Oscillations (BAO)**

Provides measurements of the angular diameter distance $D_A$(z) and/or the Hubble rate H(z) through the BAO scale in galaxy clustering. Radial BAO directly constrains H(z) in discrete redshift bins, while transverse BAO constrains $D_A$(z). Large-scale structure surveys typically publish $H(z_i)$, $D_A(z_i)$ and a covariance matrix for those points.

**Cosmic chronometers**

Provides direct, nearly model-independent estimates of $H(z)$ at specific redshifts *via* differential aging of passively evolving galaxies $(H(z) = -(1+z)^{-1}\, dz/dt)$. These are usually given as $H^{obs}(z_j) \pm \sigma H_{,j}$ and can be incorporated as Gaussian likelihood terms.

Optionally, one may include a prior on $H_0$ (the $z \longrightarrow 0$ limit of $H(z)$) or on $\Omega_{m,0}$ from CMB+BAO fits. The point of SDG, however, is that we do not assume a fundamental constant Λ. We only assume homogeneity and isotropy at large scales, which is the same assumption entering the standard background fits [5–7].

## Model parameterization

To make the likelihood computable we choose a minimal parameterization of the background expansion across the redshift range of interest ($0 \lesssim z \lesssim 2$). There are two natural parameterizations:

**Parameterization A (GR+ΛCDM-like baseline)**

Assume

$$H^2(z) = H_0^2\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0}),$$

with free parameters H0, $\Omega_{m,0}$. This is the standard spatially flat ΛCDM background. Here $\beta(z)$ is not an independent parameter; the model implies $\beta(z)$ const at low z through Eq. (127).

Parameterization B (SDG-like). Allow a mild redshift dependence in the acceleration sector while preserving matter dilution at high z. A convenient two-parameter deformation is

$$H^2(z) = H_0^2\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})\Xi(z).$$

where

$$\Xi(z) = 1 + \epsilon_1 z + \epsilon_2 z^2.$$

Here ϵ1 and $\epsilon_2$ capture departures from a constant late-time acceleration. Crucially, $\Xi(z)$ is not interpreted as a new dark-energy fluid with an imposed equation of state; instead, it is taken as a phenomenological stand-in for the evolving background-density coupling β(z) that appears in SDG. The parameter set is now $\{H_0, \Omega_{m,0}, \epsilon_1, \epsilon^2\}$.

For each trial parameter set, we construct:

- $H(z)$ directly from the expression above,
- comoving distance $\chi(z) = \int_0^z dz'/H(z')$,
- angular diameter distance $D_A(z) = \chi(z)/(1+z)$,
- luminosity distance $D_L(z) = (1+z)^2 D_A(z)$,
- $\rho_{bg}(z) = 3H^2(z)/(8\pi G)$
- $\beta(z)$ from Eq. (127),
- and, if desired, $q(z)$ from $q(z) = -1 - d\,ln\,H/d\,ln(1+z)$ to cross-check Eq. (129).

## Likelihood construction

Once $H(z)$, $D_A(z)$ and $D_L(z)$ are predicted for a given parameter set, we evaluate the likelihood as follows:

**Supernovae:**

$$-2ln\mathcal{L}_{SN} = \Delta\mu^T C_{SN}^{-1}\Delta\mu$$

where Δμ is the vector of differences between observed and predicted distance moduli $\mu(z_k) = 5log_{10}[D_L(z_k)/10pc]$ and $C_{SN}$ is the published covariance matrix.

**BAO:**

$$-2ln\mathcal{L}_{BAO} = \Delta d^T C_{BAO}^{-1}\Delta d$$

where $\Delta d$ is the vector of residuals between observed and predicted $\{H(z_i), D_A(z_i)\}$ (or related compressed BAO combinations) and $C_{BAO}$ is the BAO covariance matrix.

**Cosmic chronometers:**

$$-2ln\mathcal{L}_{CC} = \sum_j \frac{[H_{obs}(z_j) - H_{model}(z_j)]^2}{2\sigma^2 H_{,j}}$$

treating the chronometer points as independent Gaussians.

The total likelihood is

$$\ln\mathcal{L}_{tot} = ln\mathcal{L}_{SN} + ln\mathcal{L}_{BAO} + ln\mathcal{L}_{CC} + ln\,\mathcal{L}_{prior},$$

where ln $\mathcal{L}_{prior}$ can impose broad physical priors such as $\Omega_{m,0} \in (0, 1)$ and $H_0 > 0$.

## Model comparison: ΛCDM consistency *vs* SDG-like evolution

The likelihood analysis proceeds in two fits:

- Fit Parameterization A (ΛCDM-like, no $\epsilon_i$). This yields best-fit $H_0$, $\Omega_{m,0}$ and an implied "constant" $\beta(z)$ with error bars propagated from the covariance of the fitted parameters.

- Fit Parameterization B (SDG-like, with $\epsilon_1$, $\epsilon_2$ free). This yields best-fit $H_0, \Omega_{m,0}, \epsilon_1, \epsilon_2$ and therefore a reconstructed function β(z) with uncertainties.

We then ask two questions:

**Is $\epsilon_1 = \epsilon_2 = 0$ statistically allowed?**

If yes, then current data are consistent with a constant acceleration sector (i.e. GR+ΛCDM remains viable and simplest SDG is constrained). If no, then the data prefer mild redshift evolution in the acceleration sector, which in GR+ΛCDM cannot be accommodated without adding an explicitly dynamical dark-energy component. In SDG this evolution is expected, because β is determined by $H(z)$ and $\rho_{\mathrm{bg}}$(z) and can vary with epoch.

**Is $\frac{d\beta}{dz}$ at $z \lesssim 1$ consistent with zero within the ΛCDM fit and significantly nonzero in the SDG-like fit?**

This is the direct implementation of Eq. (128). A statistically significant detection of $\frac{d\beta}{dz} \neq 0$ at late times rules out "pure constant-Λ" and is naturally interpreted as evidence for SDG-like behaviour.

Formally, one can compare the two fits using standard information criteria (AIC/BIC) or a Bayes factor. The important physical point is that the observable being compared is not an exotic new field: it is the redshift evolution of the effective background-density coupling β(z) that sources acceleration.

**Outcome and significance**

This pipeline converts real low-redshift background cosmology data (SNe Ia, BAO, cosmic chronometers) into a statistically testable statement:

- "The effective acceleration sector is consistent with a constant Λ" (favouring GR+ΛCDM),
- or "The effective acceleration sector must evolve with redshift at late times" (supporting the SDG interpretation in which β is a dynamical background-density coupling rather than a fundamental constant).

In particular, because SDG ties β directly to $(H, \rho_{bg}, q)$ through Eqs. (127) and (129), any statistically significant late-time drift in β(z) is not an optional extra assumption of SDG — it is a built-in prediction. Conversely, if β(z) is observationally indistinguishable from a constant across $0 \lesssim z \lesssim 1$, then SDG is forced into a regime where it mimics ΛCDM at the background level and can be correspondingly constrained.

This provides a clean, likelihood-level discriminator between SDG and GR+ΛCDM using existing classes of cosmological data.

## Appendix D: Methods for Test B (Compact-Object Ringdown and Horizon-Scale Observables)

This appendix outlines a complementary observational test of SDG in the strong-field regime. Where Appendix C targets the cosmological background, this section describes how compact-object observations—gravitational-wave ringdown and horizon-scale imaging— can probe the finite-density core predicted by SDG.

### Physical premise

In GR, the end state of a stellar collapse or binary merger is a Kerr black hole with an event horizon and a curvature singularity at r = 0. In SDG, the singularity is replaced by a finite-density core with continuous curvature. The external geometry asymptotically approaches the Schwarzschild or Kerr form, but the absence of a true horizon allows partial reflection of gravitational waves and small deviations in photon orbits.

### Ringdown spectrum

After a merger, the remnant emits damped oscillations characterized by Quasi-Normal Modes (QNMs). To test SDG against GR, one computes and compares the QNM spectra.

**Procedure**

1. Solve the axial and polar perturbation equations for the static SDG metric:

$$\frac{d_2\Psi_\ell}{dr_*^2} + \left[\omega^2 - V_\ell^{SDG}(r)\right]\Psi_\ell = 0,$$

where $r_*$ is the tortoise coordinate and $V_\ell^{SDG}(r)$ is the effective potential obtained from the SDG field equations. In the GR limit $V_\ell^{SDG} \to V_\ell^{Schw}$.

2. Impose outgoing-wave boundary conditions at spatial infinity and regular (finite) conditions at r = 0 instead of the GR condition at the event horizon.
3. Use a Leaver-type continued-fraction method or WKB approximation to find the complex frequencies $\omega_{n\ell}$.
4. Compare the fundamental mode $\omega_{20}$ and overtones with those measured in LIGO/Virgo/KAGRA events.

**Expected discriminant:** SDG predicts slightly lower damping (larger quality factor $Q = Re\,\omega/2|Im\,\omega|$) and possibly secondary "echo" pulses in the time domain, arising from partial reflection at the finite-density core. GR predicts pure exponential decay with no echoes. Detecting such echoes or frequency shifts beyond measurement uncertainty would constitute direct evidence of SDG's finite-core structure.

### Horizon-scale imaging

For a rotating SDG compact object, the external metric can be expressed as a Kerr-like solution with a modified lapse function $f_{SDG}(r)$ that differs from Kerr near the would-be horizon. This modifies the photon-sphere radius $r_{ph}$ and thus the apparent shadow diameter observed by the Event Horizon Telescope (EHT).

**Procedure**

1. Integrate null geodesics for photons with impact parameter b = L/E in the equatorial plane of the SDG metric.
2. Determine the critical $b_{ph}$ at which photons execute unstable circular orbits.
3. Compute the corresponding angular shadow radius $\theta_{sh} = b_{ph}/D$, where D is the source distance.
4. Compare $\theta_{sh}$ and ring substructure with EHT measurements of M87* and Sgr A*.

**Expected discriminant**

In GR, the shadow diameter for a Schwarzschild black hole is $2\sqrt{27}GM/c^2D$. SDG predicts small (percent-level) corrections due to $f_{SDG}(r)$ near the core. Consistency with observed EHT diameters constrains those corrections and hence the allowed central density $\rho_c$. A statistically significant deviation from the Kerr prediction would support SDG.

## Simulation workflow

A practical numerical study would proceed as follows:

- Adopt a parameterized form of the SDG metric function $f_{SDG}(r)$ consistent with the regular interior solution of Sec. 11.
- Compute $V_\ell^{SDG}(r)$ and $f_{SDG}(r)$ analytically or numerically.
- Generate synthetic waveforms using the Einstein Toolkit or a simplified time-domain evolution code and process them with LIGO/Virgo open-data pipelines to search for late-time echoes.
- Perform raytracing simulations of photon trajectories for a range of spins a/M and compare the resulting shadow sizes to EHT data.

## Interpretation

Agreement with GR predictions (no echoes, Kerr-consistent shadows) would constrain SDG parameters $(\rho_c, \beta)$ and verify that deviations are small in the observed regime. Detection of persistent echoes or measurable departures from Kerr imaging would provide direct evidence for SDG's finite-density interiors and dynamic background coupling. Together with the cosmological β(z) analysis of Appendix 13, these strong-field tests establish a comprehensive observational program for SDG.

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## Ethical approval

This study did not involve human participants or animal experiments; therefore, formal ethical approval was not required.

## Consent to participate

No human participants were involved in this work; consent to participate is not applicable.

## Consent to publish

This manuscript does not contain any individual person's data in any form; consent to publish is therefore not applicable.

## Author Contributions

S.G.P. (Sadid Gagi Peskovi´c) conceived the theoretical framework, derived all equations and wrote the manuscript. B.B. (Bendik Bø) contributed through conceptual discussions and provided constructive insights that helped refine the theoretical development and interpretation. All authors reviewed and approved the final manuscript.

## Conflict of interest

The author declares no conflict of interest.

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